QWIRE:

A core language for quantum circuits

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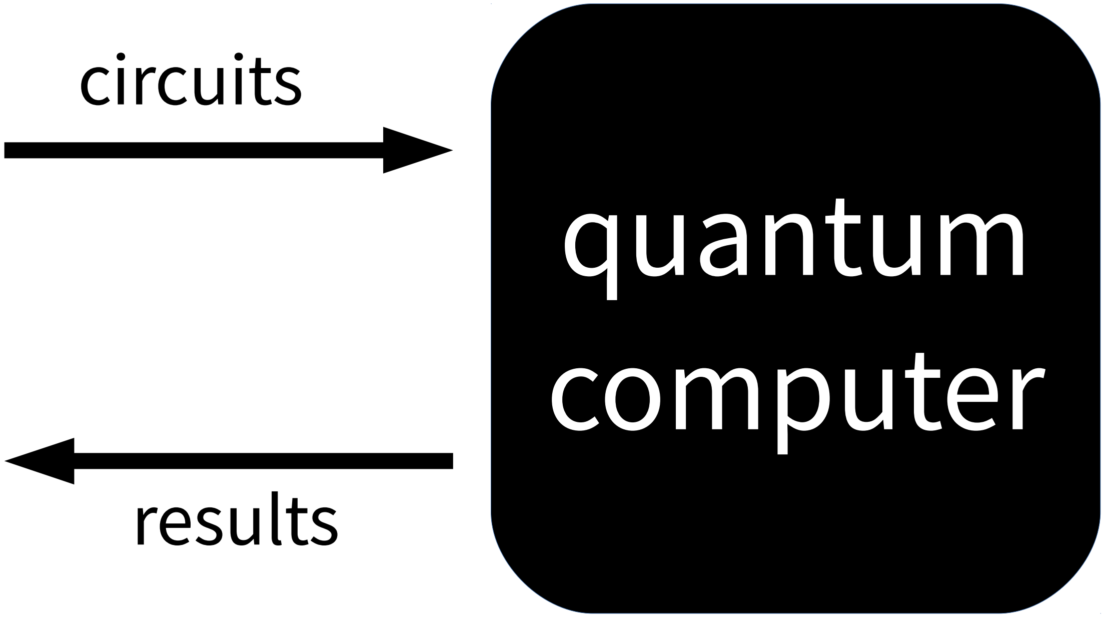
University of Pennsylvania

POPL 2017, Paris, France

The Circuit Model



circuits

quantum computer

results

Building Blocks of Quantum Computing

Building Blocks of Quantum Computing

qubits $|0\rangle$ or $|1\rangle$ or $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

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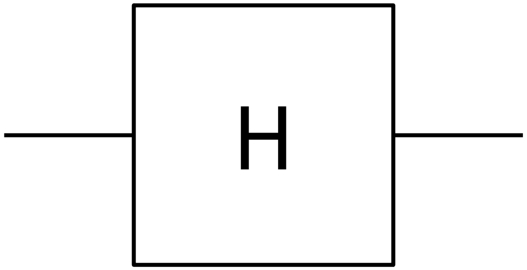
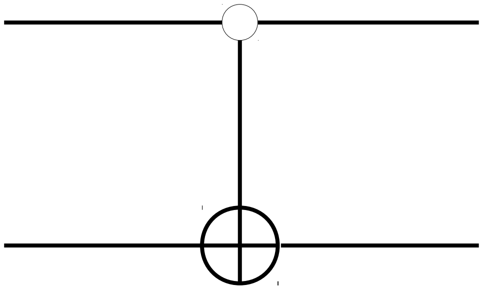


Hadamard



controlled not (CNOT)

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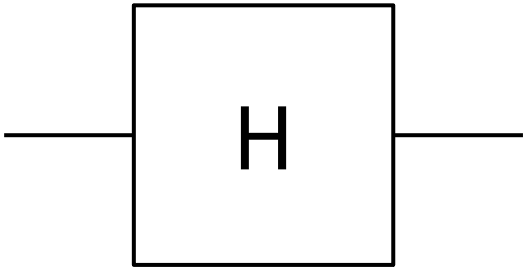

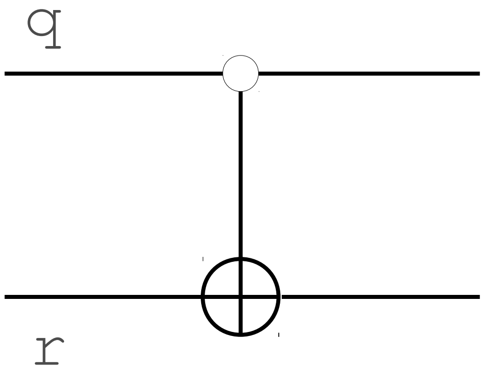

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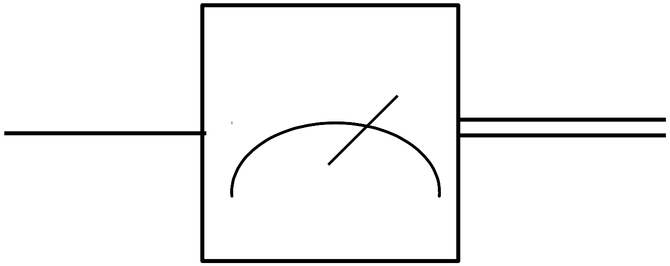

measurement

Building Blocks of Quantum Computing

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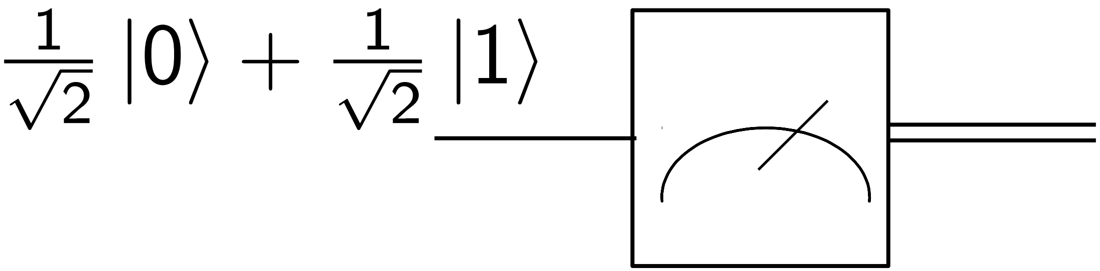

$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

measurement

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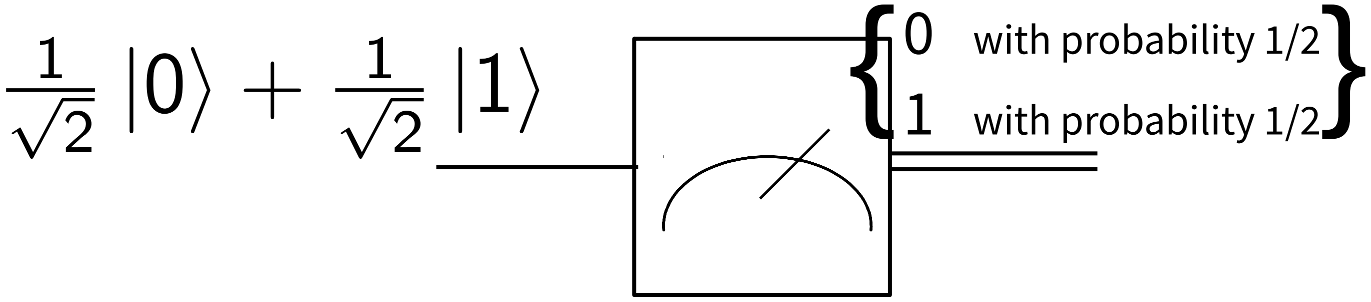


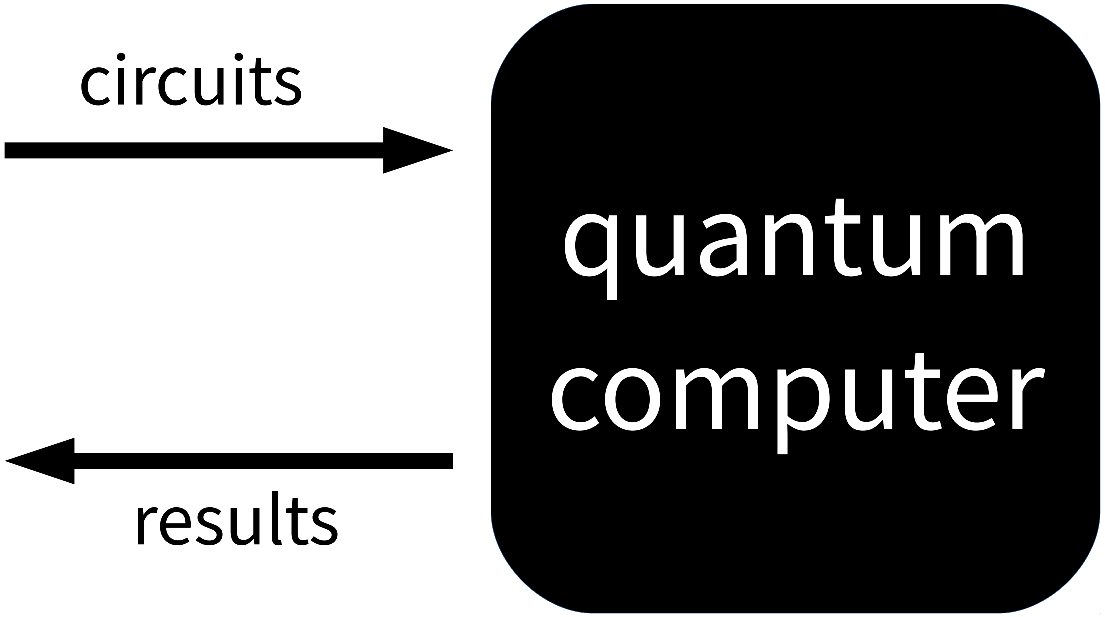
$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$\left\{\begin{matrix} 0 & \text{with probability } 1/2 \\ 1 & \text{with probability } 1/2 \end{matrix}\right\}$$

measurement

The QRAM Model of Quantum Computing

Knill, 1996



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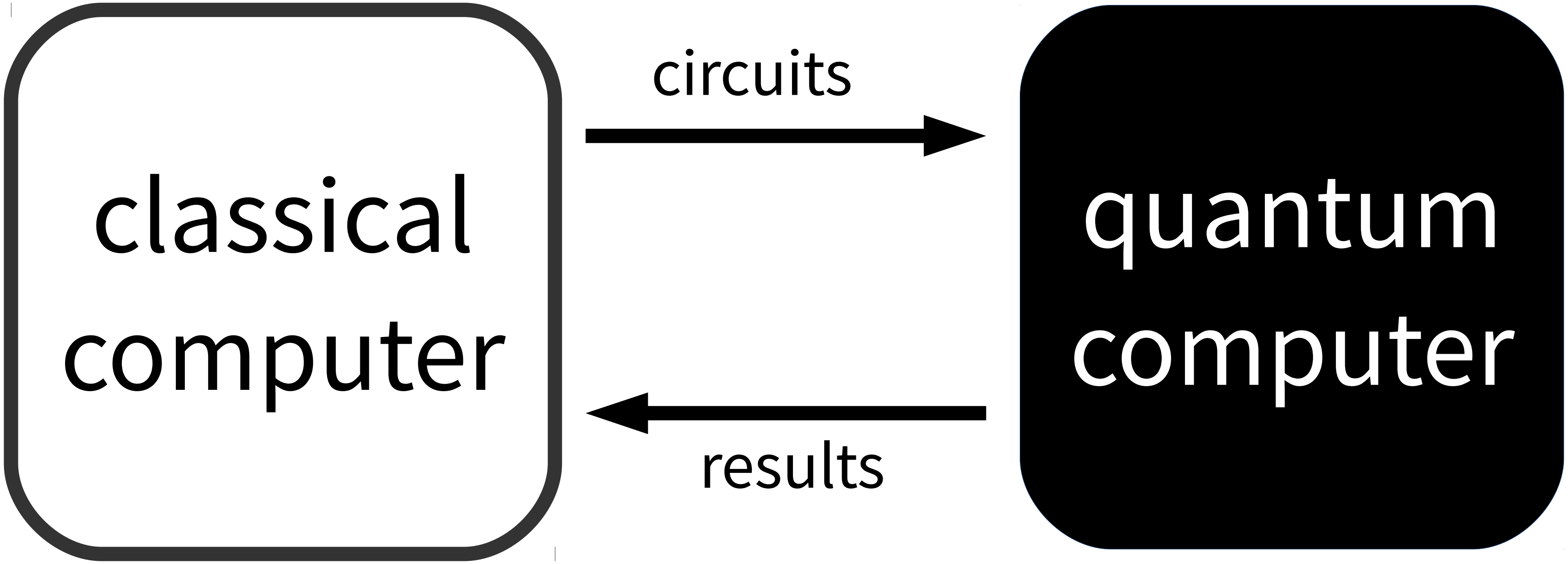

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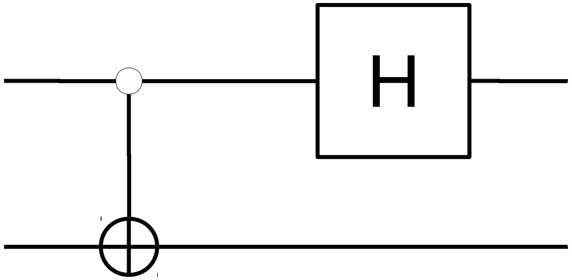
n-ary Composition



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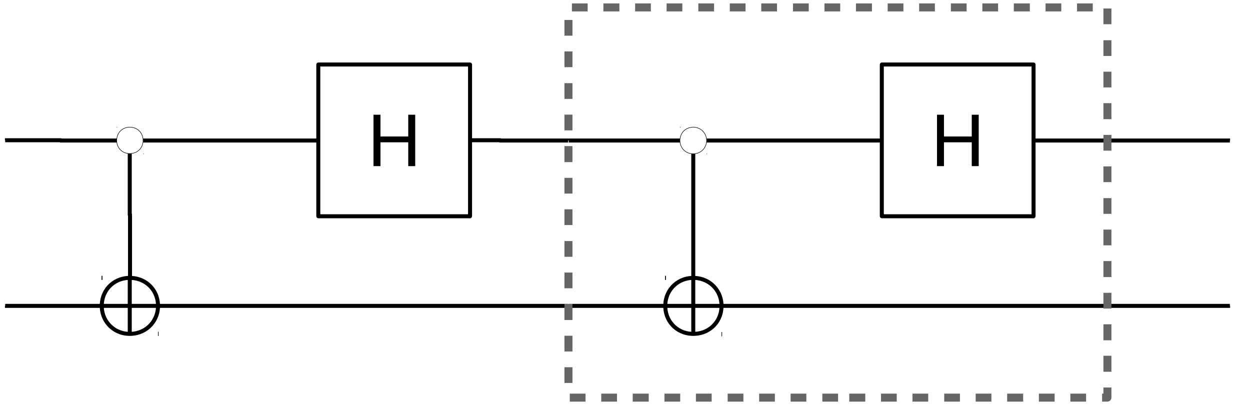

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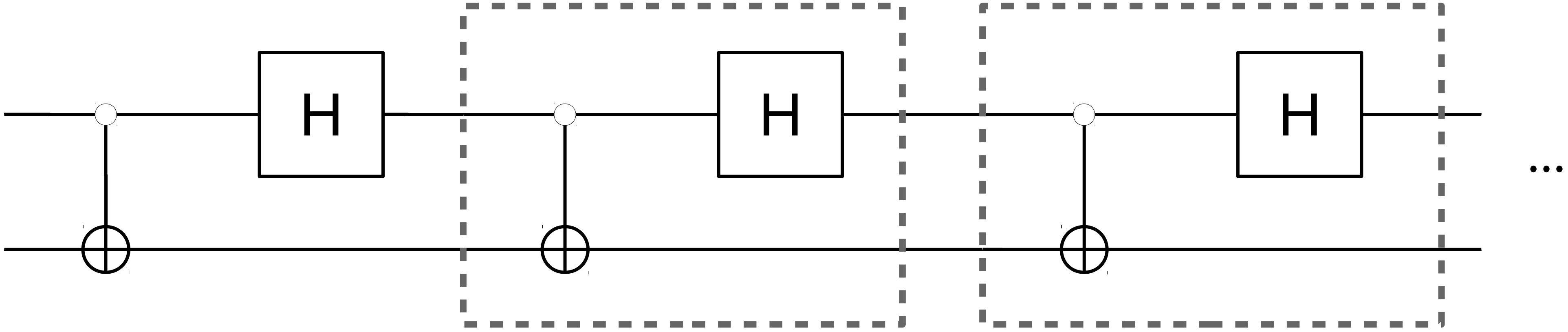

The QRAM Model of Quantum Computing

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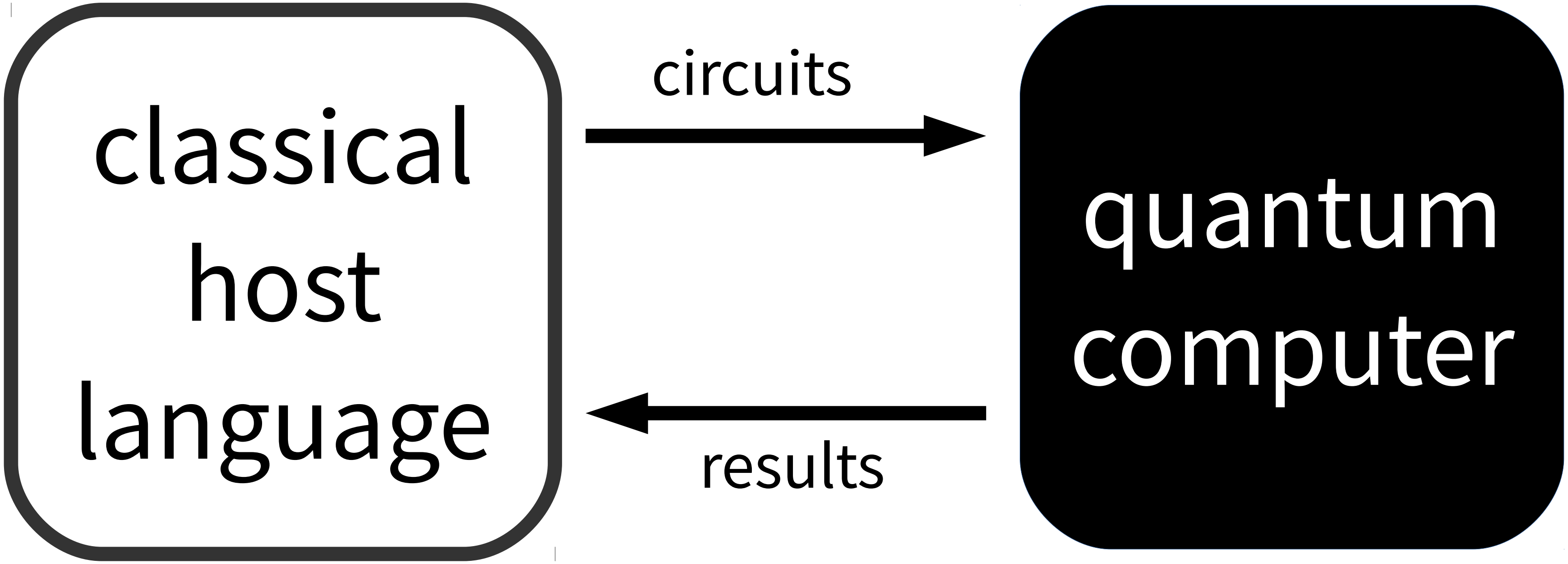
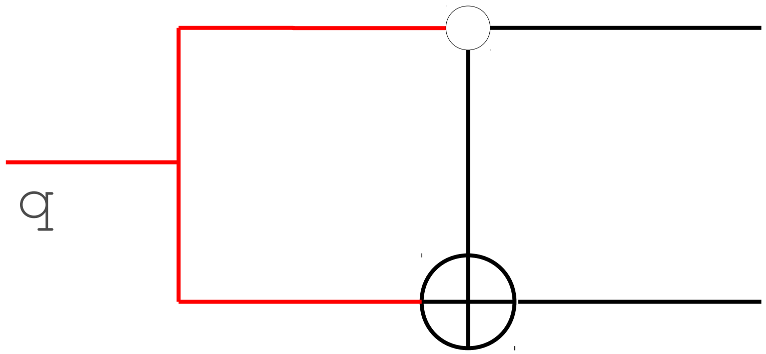


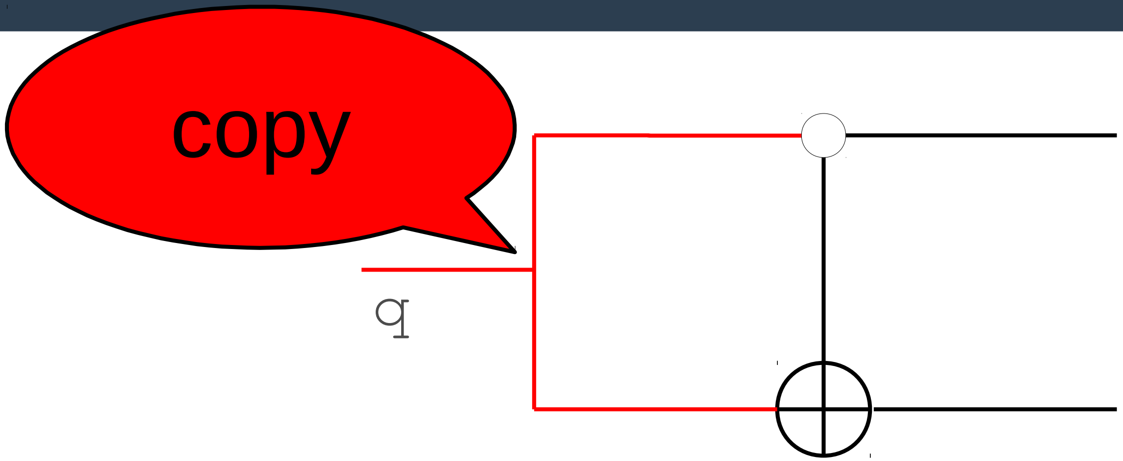
Quipper (Green et al, 2013)
LIQ$Ui|\rangle$ (Wecker and Svore, 2014)

Problem: Ill-Formed Circuits



“qif q then not q”

Problem: Ill-Formed Circuits



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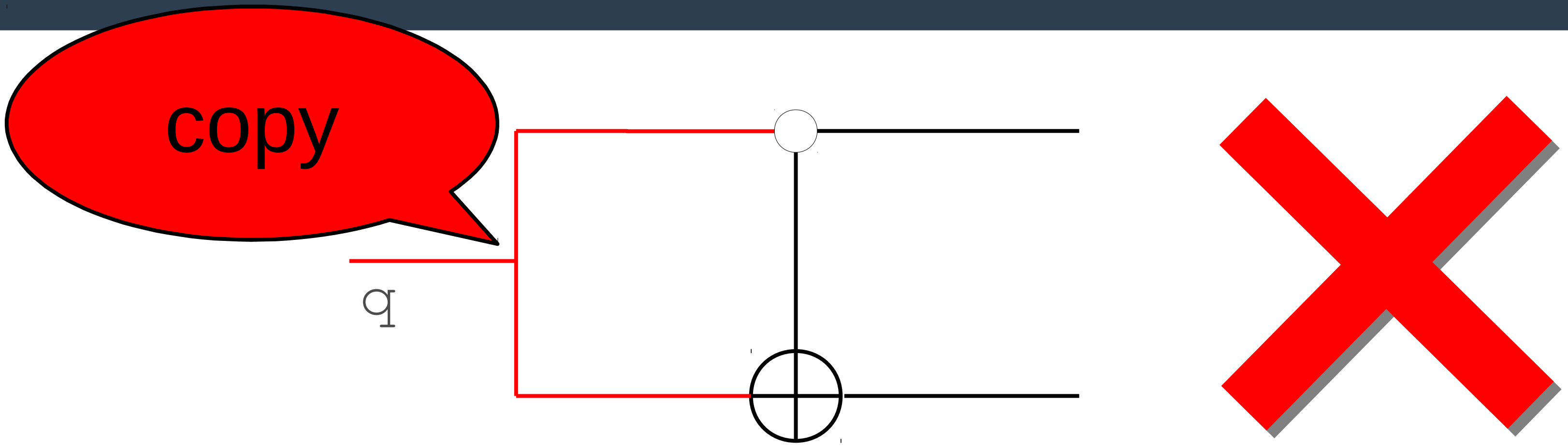
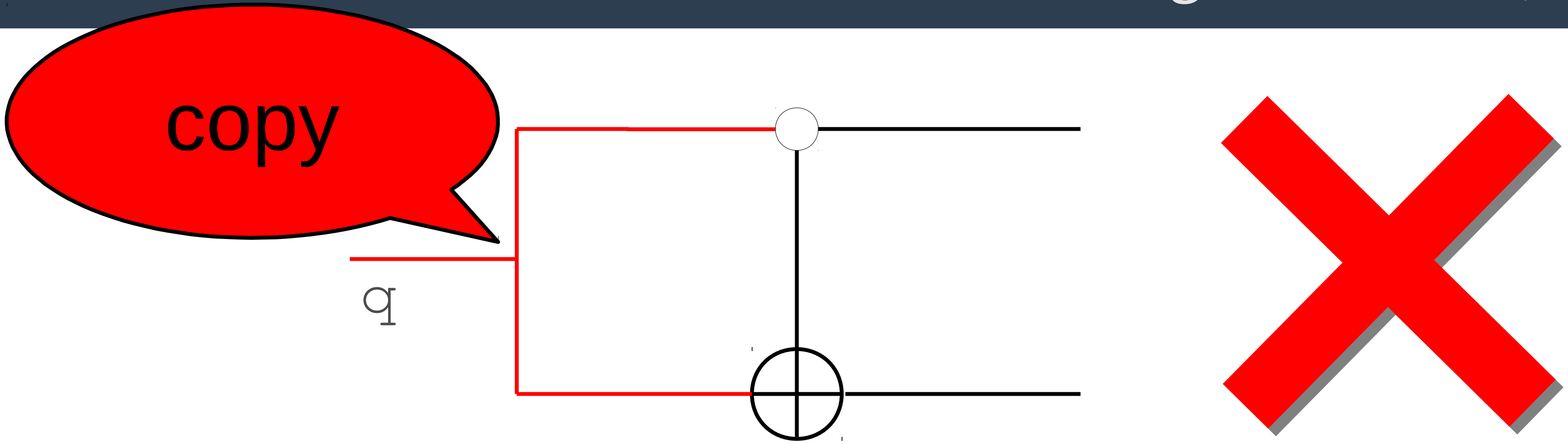


“`qif q then not q`”

no-cloning theorem: a qubit cannot be copied

Problem: Ill-Formed Circuits

Selinger & Valiron, 2009

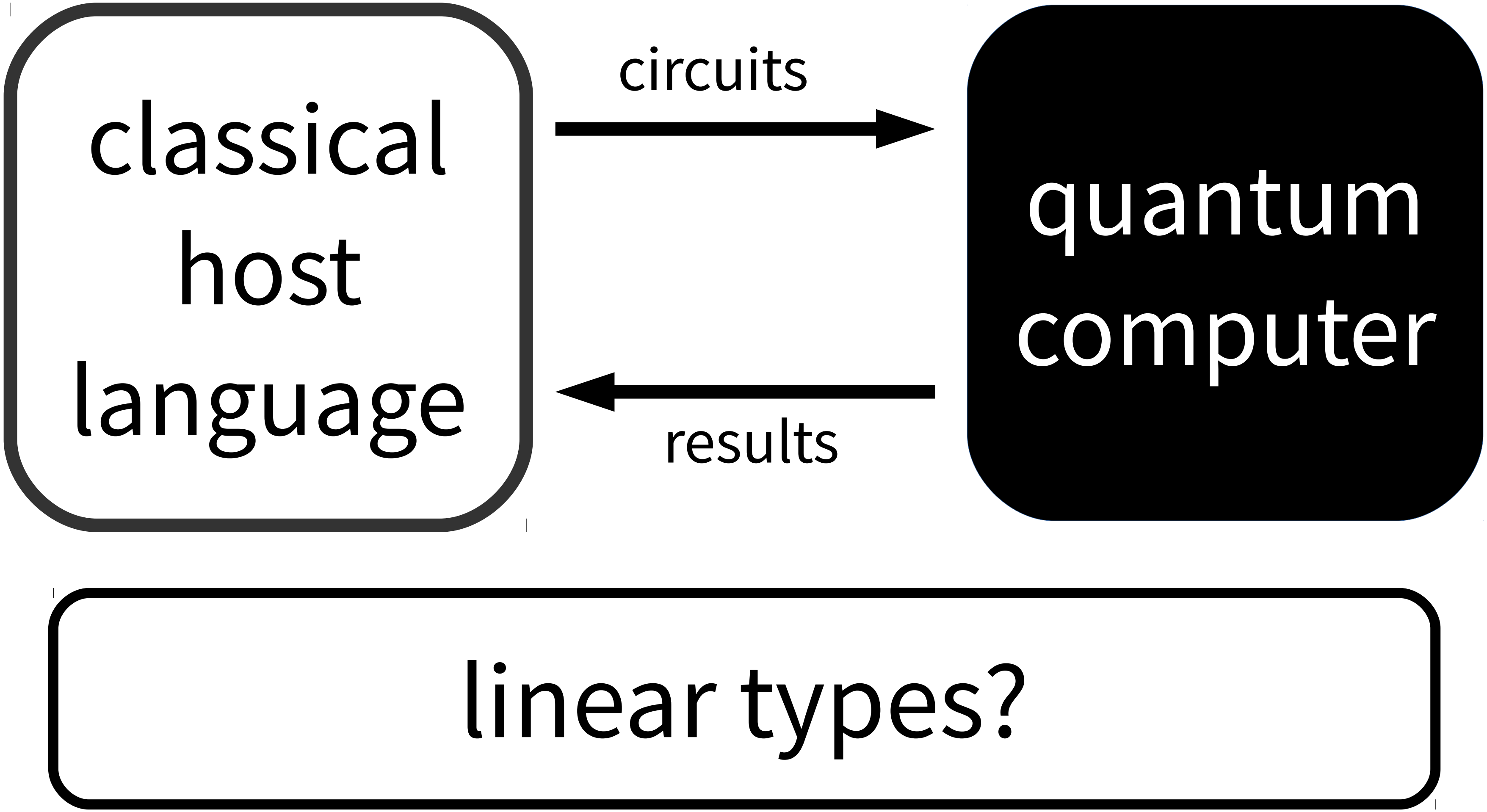


`“qif q then not q”`

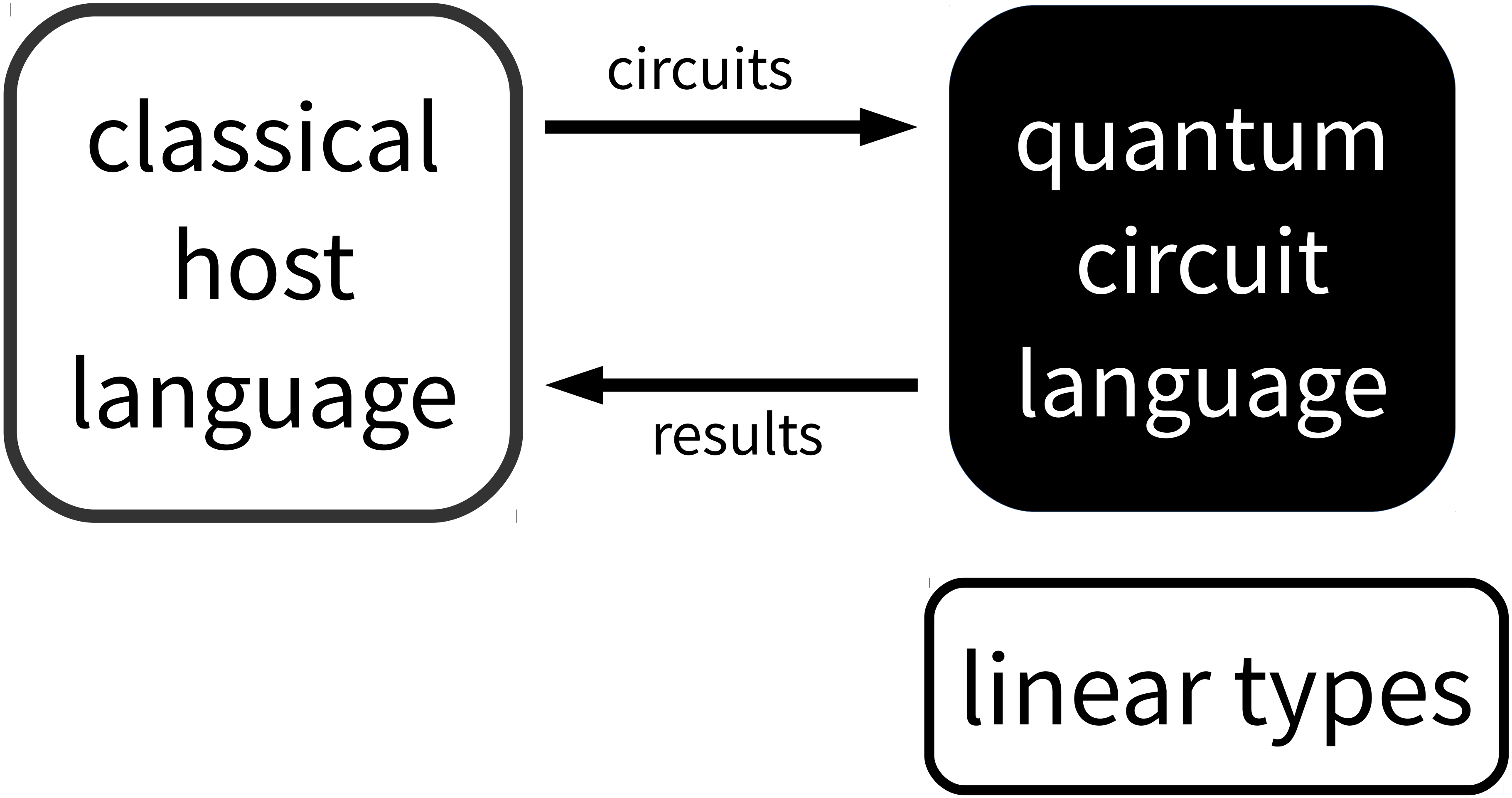
no-cloning theorem: a qubit cannot be copied

linear types ⇒ no cloning

A linear QRAM model?



Introducing $\mathcal{Q}$WIRE...



classical host language

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quantum circuit language

results

linear types

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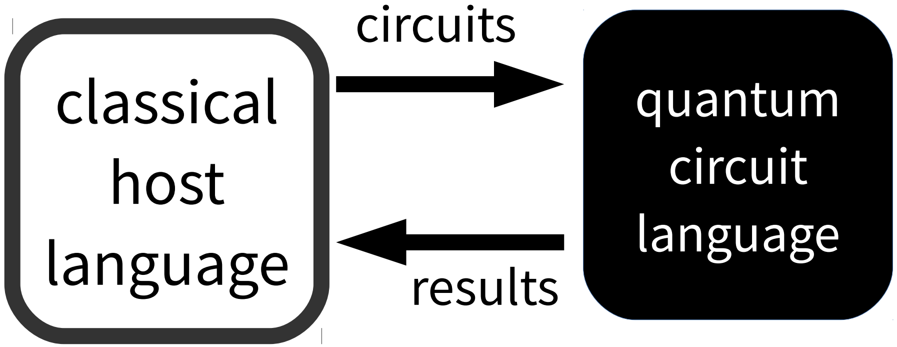

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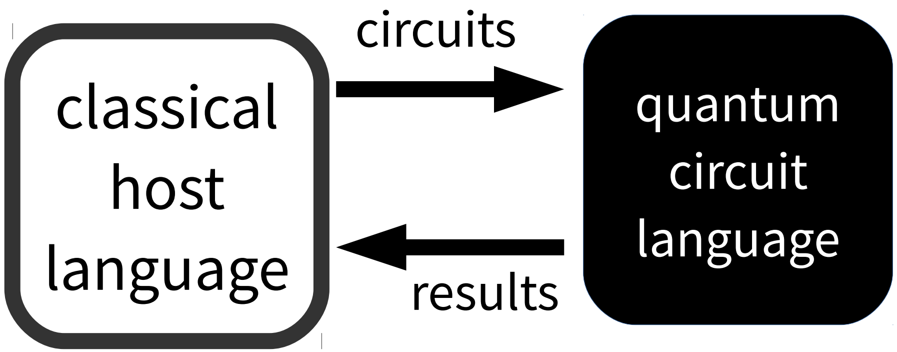
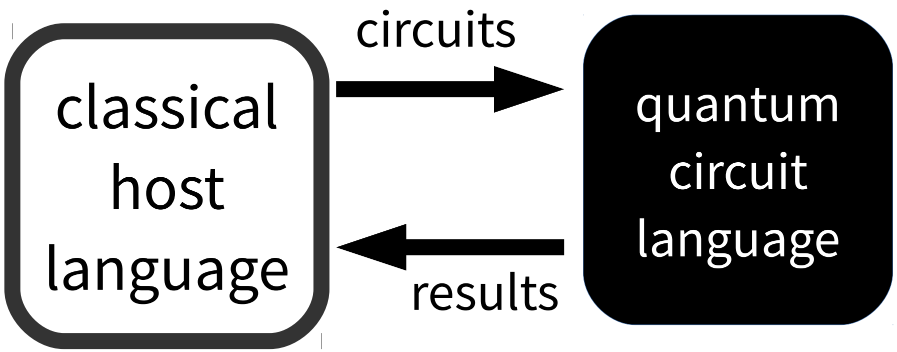


- A core language for quantum circuits

Introducing $\mathcal{Q}$WIRE…



- A core language for quantum circuits
- Safe
  - linear types for wires
  - type safety & strong normalization
  - denotational semantics

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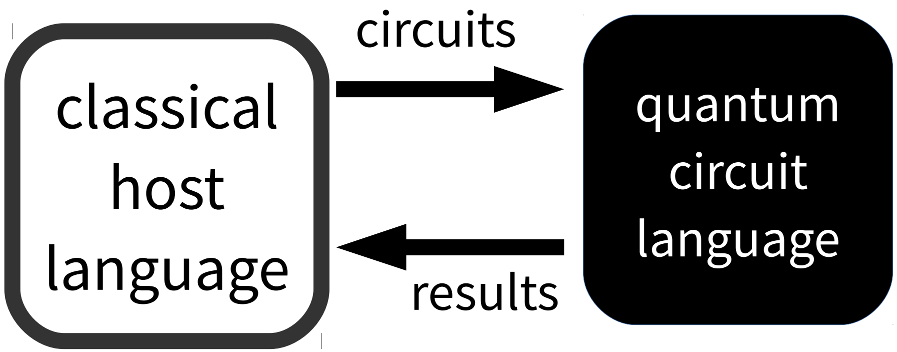
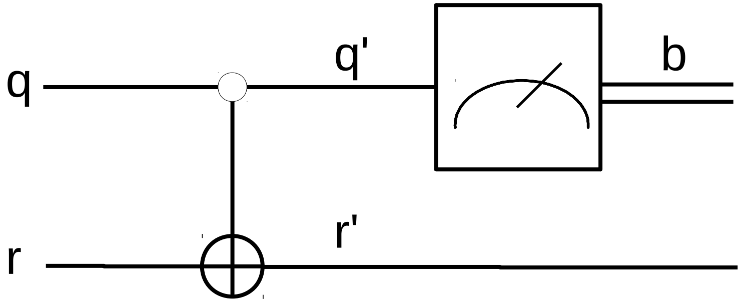


- A core language for quantum circuits
- Safe
  - linear types for wires
  - type safety & strong normalization
  - denotational semantics
- Expressive
  - structure based on the QRAM model
  - embedded in your favorite classical host language

Quantum Circuits



```
(q',r') ← gate CNOT (q,r);
b       ← gate meas q';
output (b,r')
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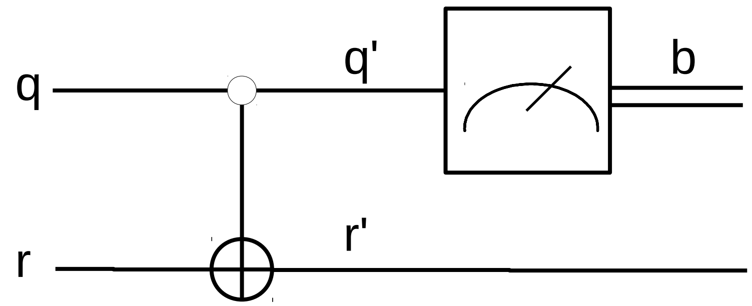
Quantum Circuits



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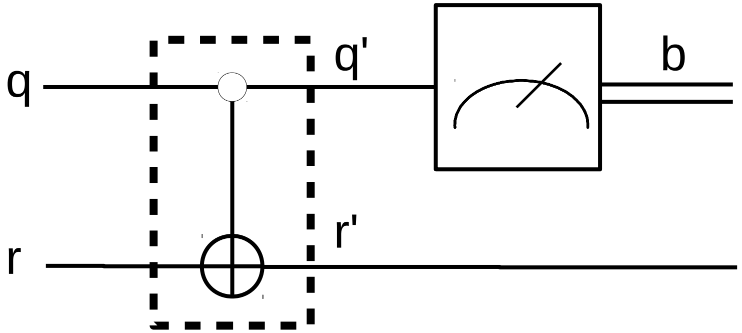

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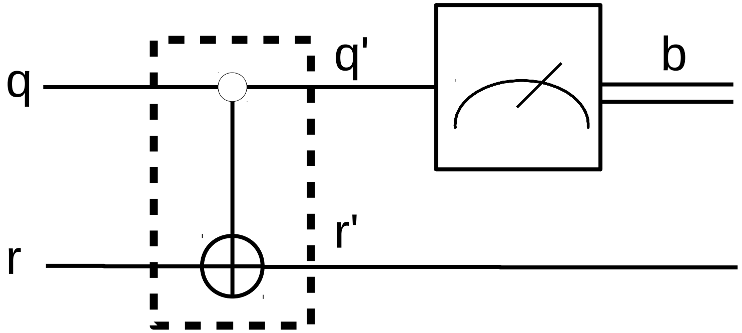
Quantum Circuits

let binding



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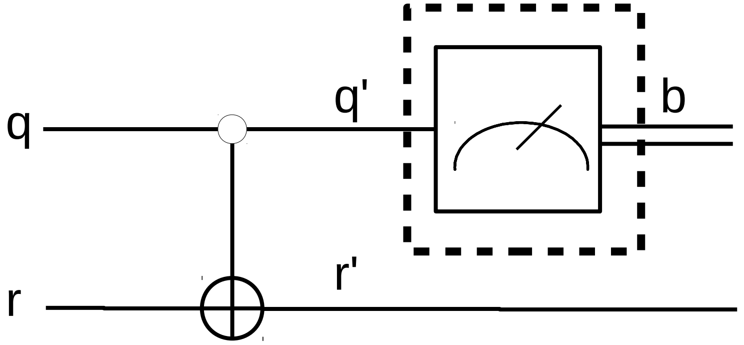

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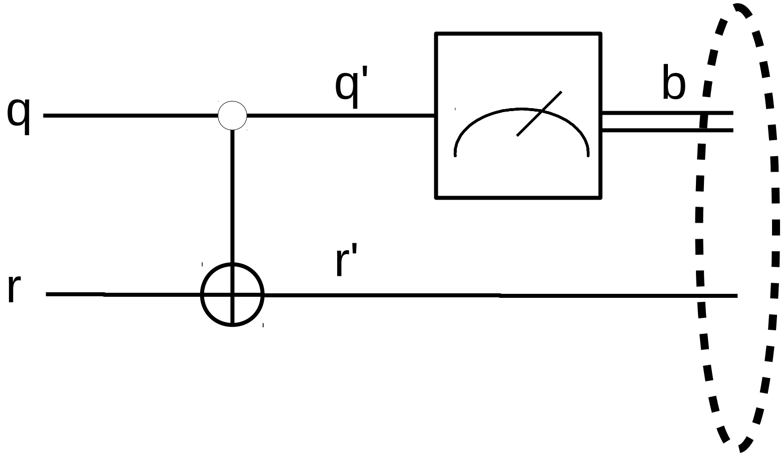

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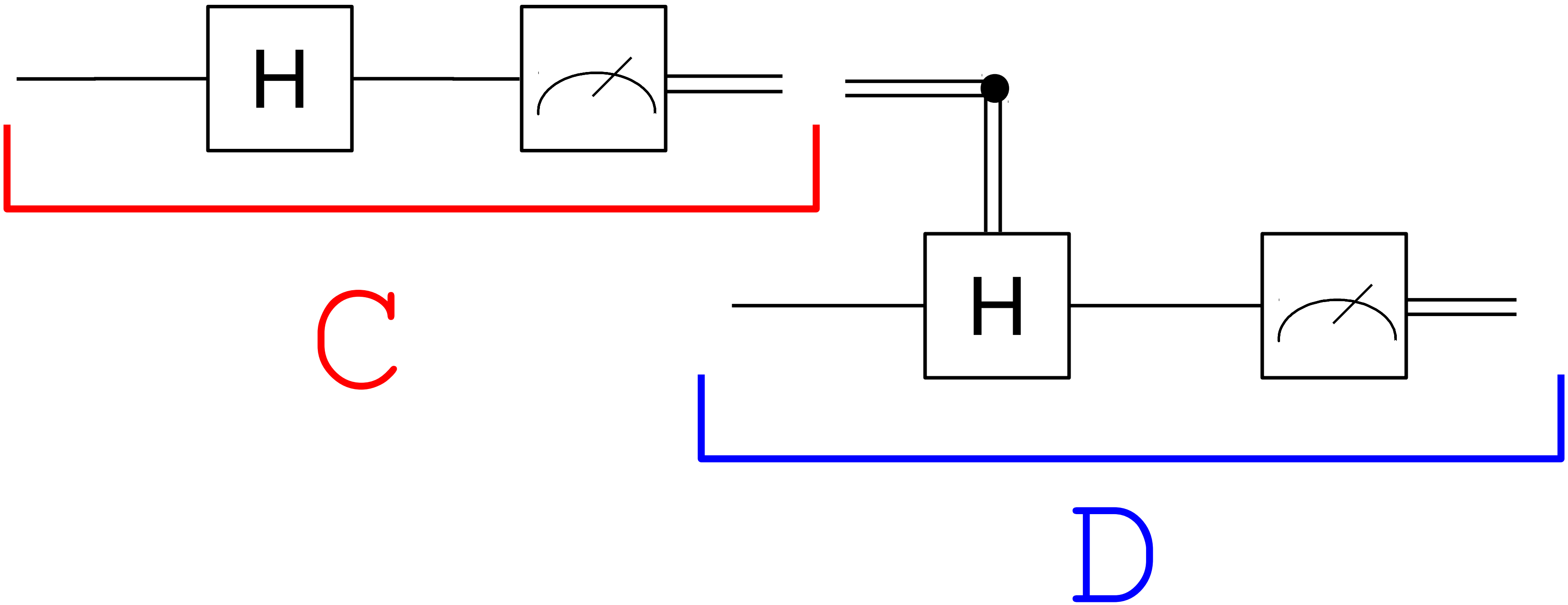

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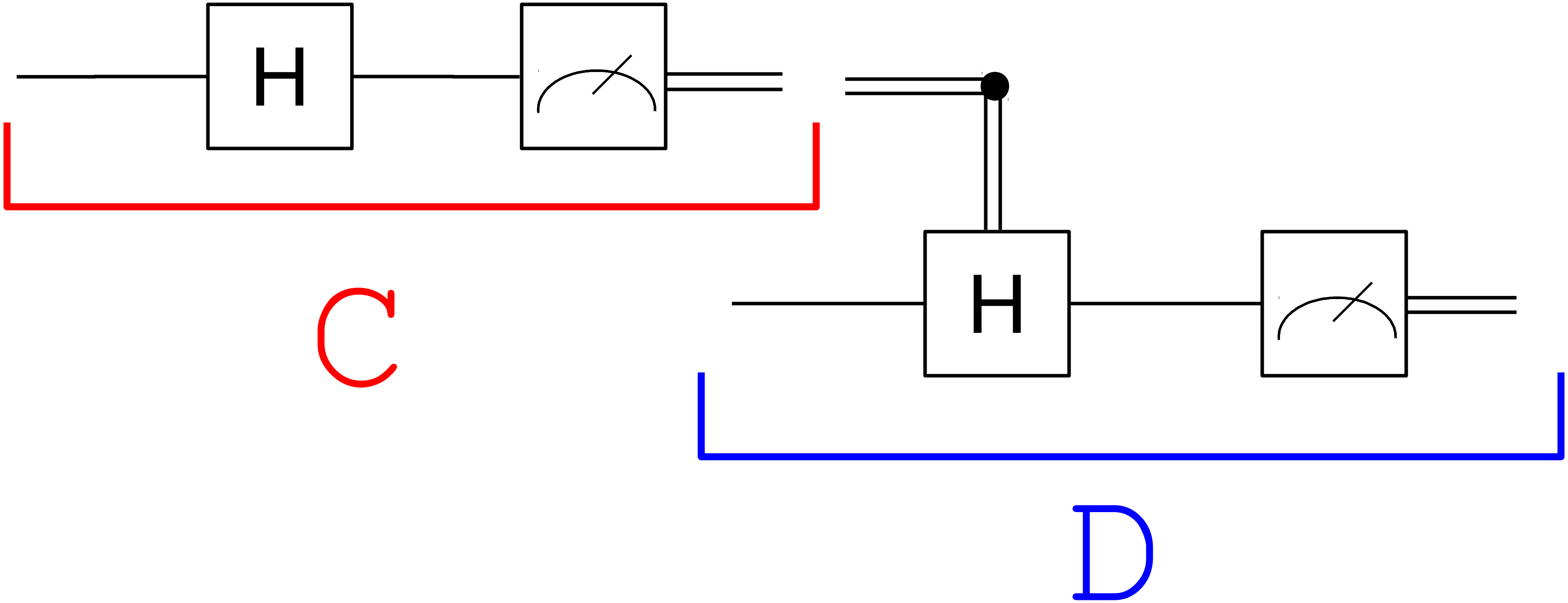
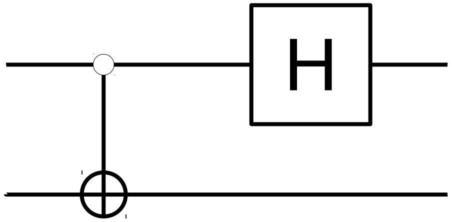


```
b ← C; D
```

n-ary Composition?



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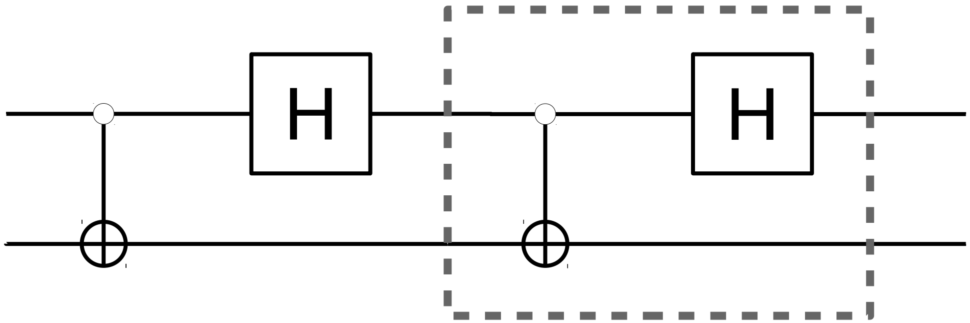

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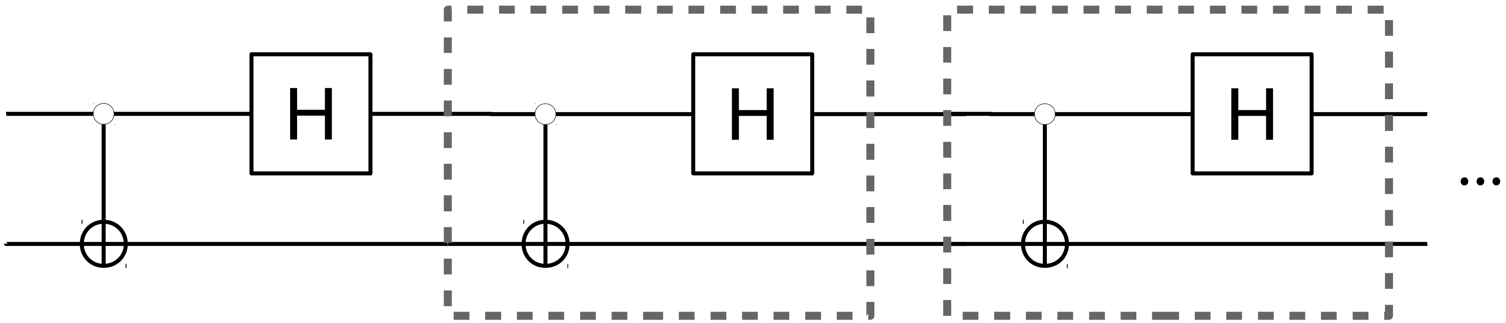

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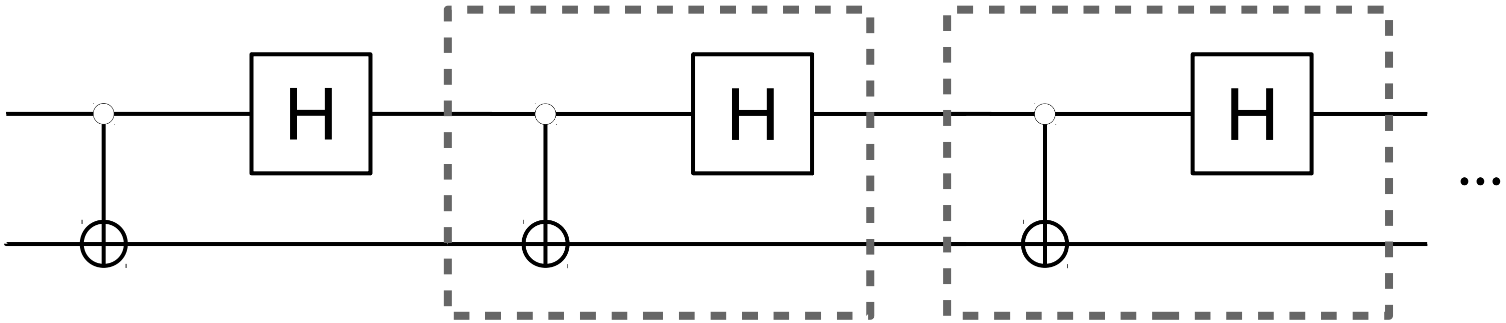

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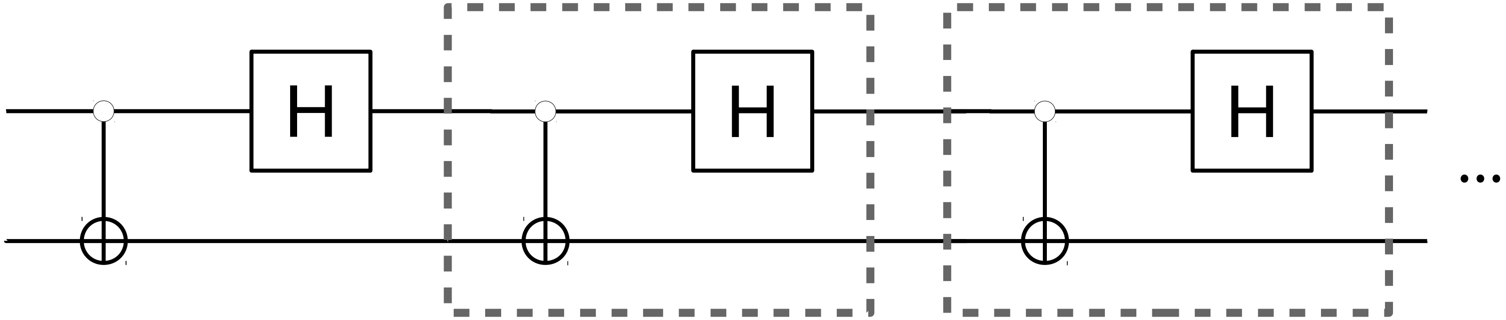

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...
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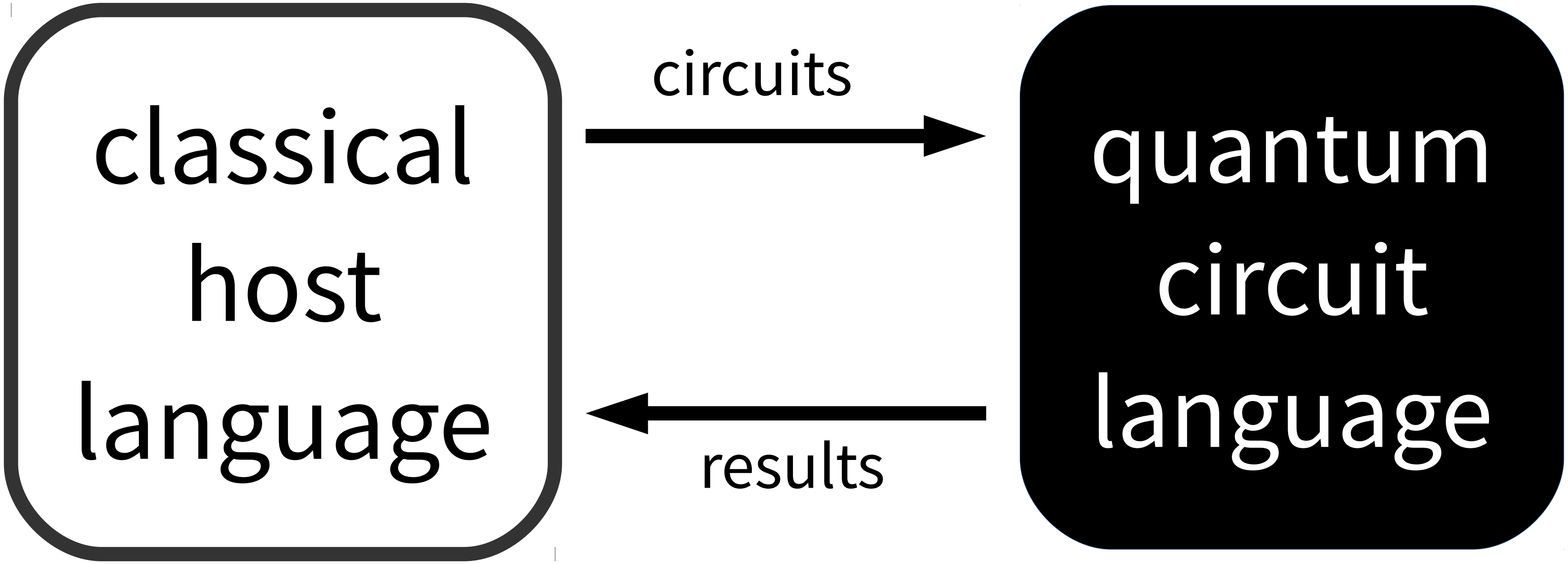
Typing Judgments in $\mathcal{Q}$WIRE



classical host language

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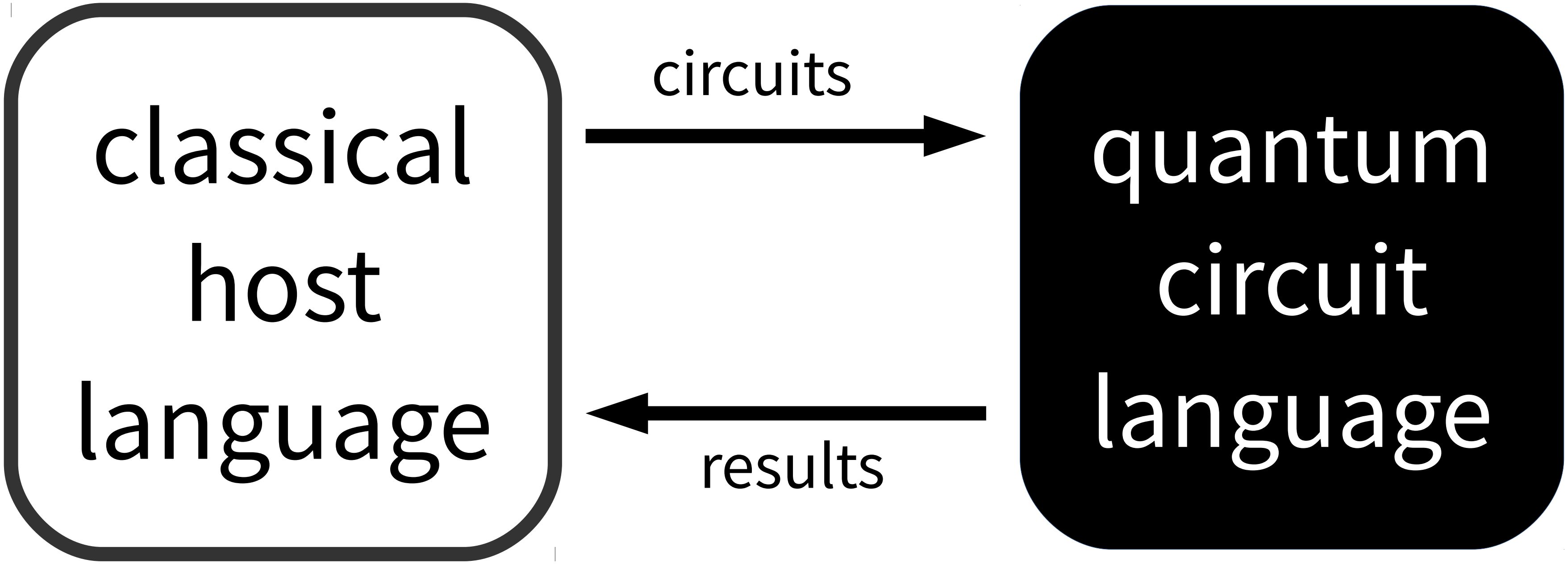
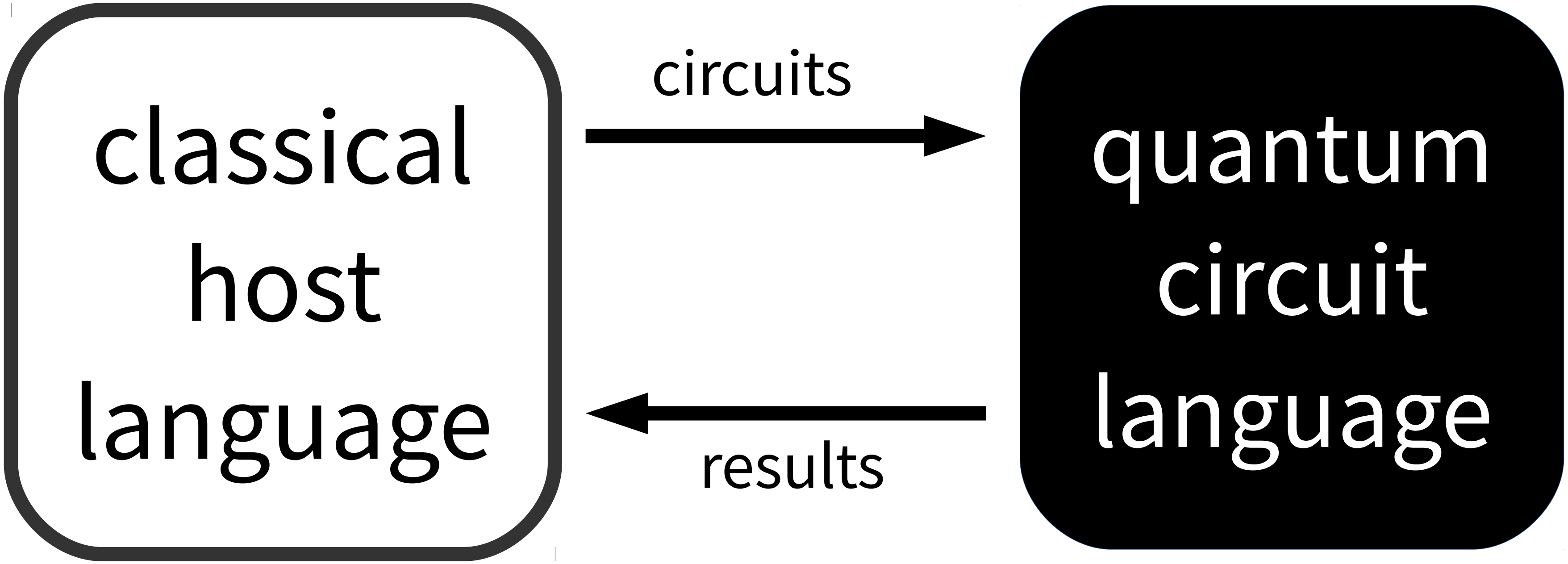
quantum circuit language

Typing Judgments in $\mathcal{Q}$WIRE



$$\Gamma \vdash t : A$$

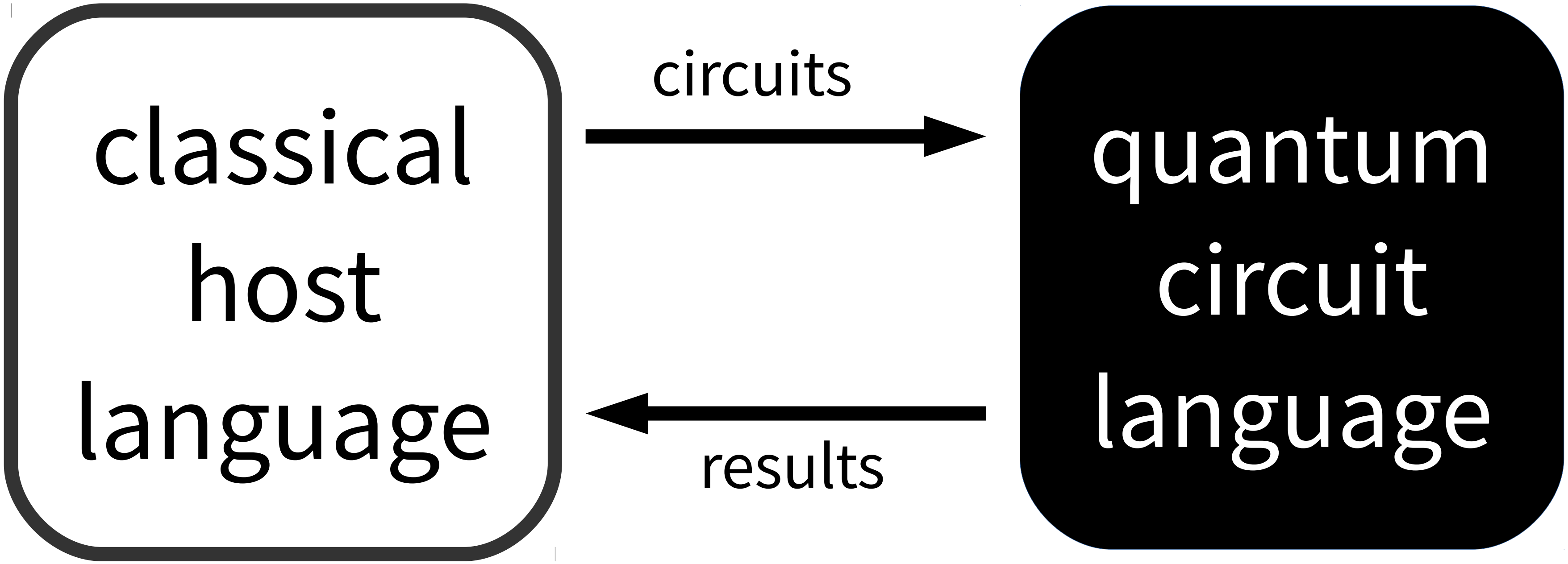
Typing Judgments in $\mathcal{Q}$WIRE



$$\Gamma \vdash t : A$$

$$\Gamma; \Delta \vdash C : W$$

Typing Judgments in $\mathcal{Q}$WIRE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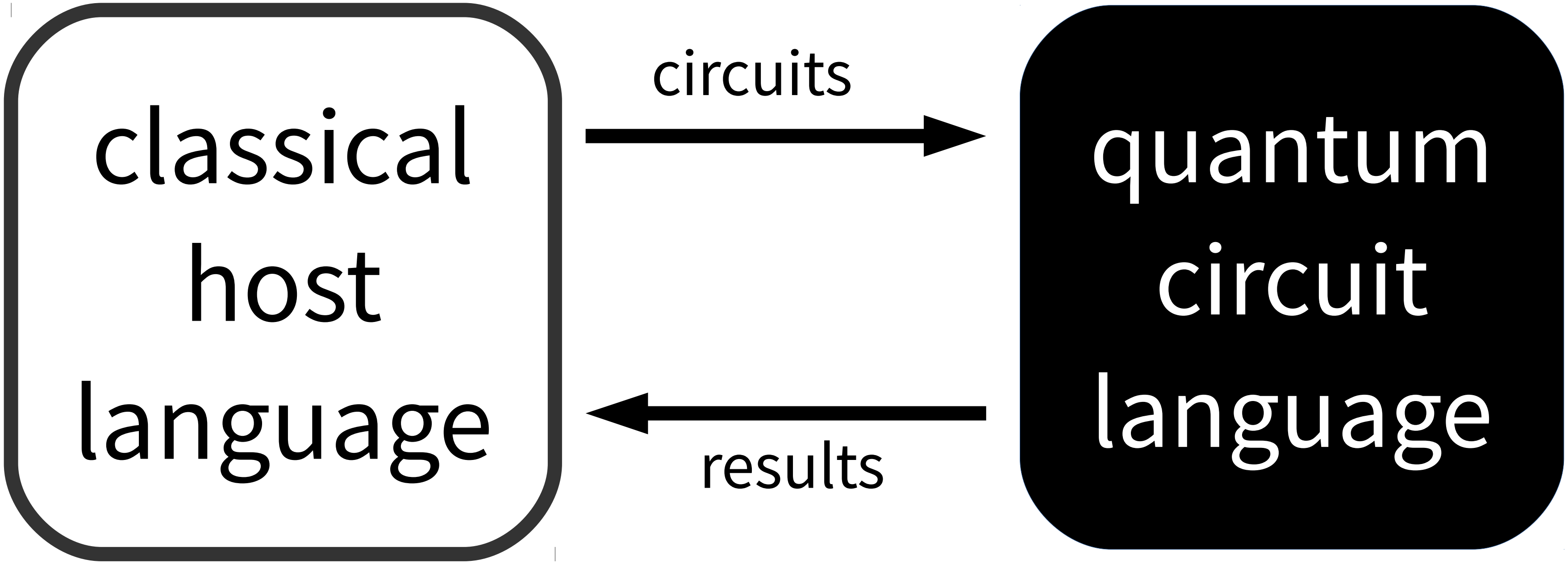
$$\Gamma \vdash t : A$$

$$\Gamma ; \underline{\Delta} \vdash C : \underline{W}$$

linear wire types

Typing Judgments in $\mathcal{Q}$WIRE

Benton, 1995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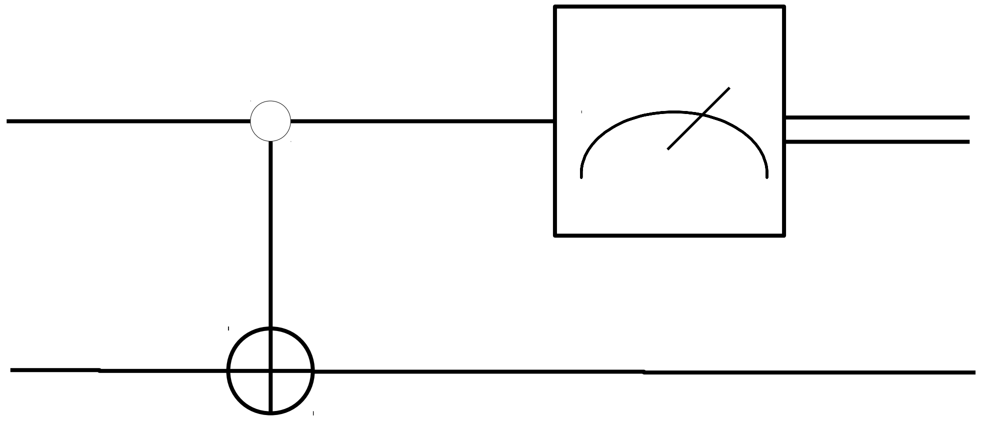
host language types

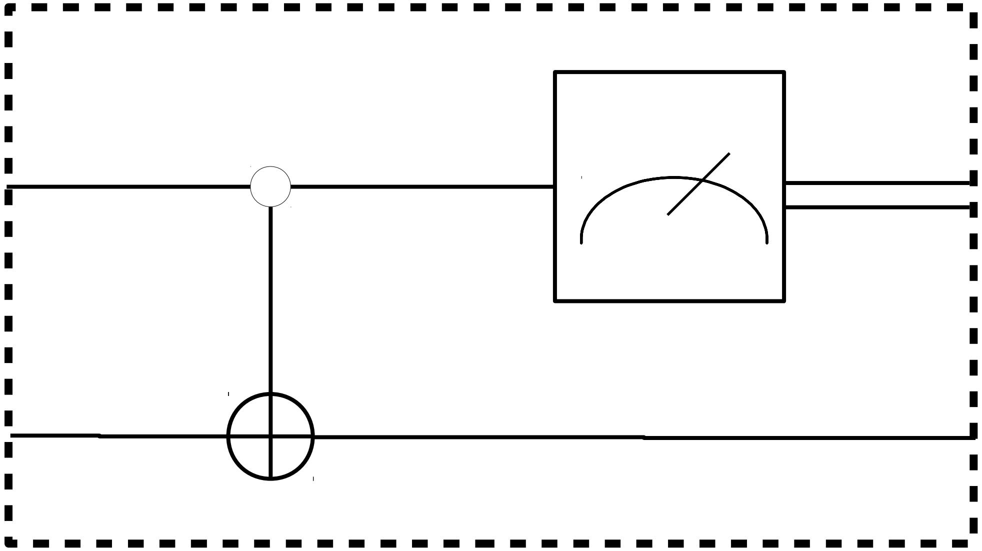
linear wire types

Boxes



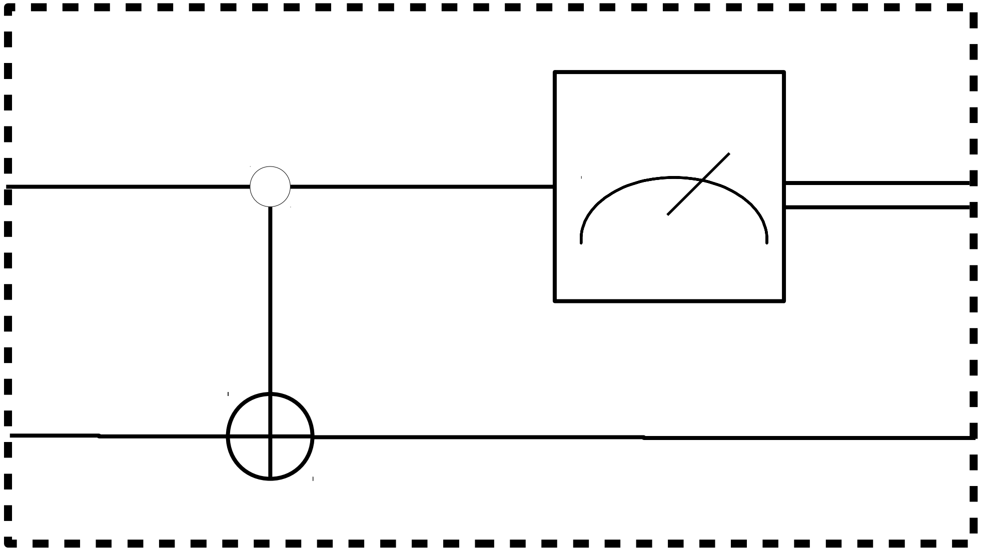
$$\Gamma; q : \texttt{qubit}, r : \texttt{qubit} \vdash C : \texttt{bit} \otimes \texttt{qubit}$$

Boxes



$$\frac{\Gamma; q : \mathtt{qubit}, r : \mathtt{qubit} \vdash C : \mathtt{bit} \otimes \mathtt{qubit}}{\Gamma \vdash \mathsf{box}\,(q,r) \Rightarrow C \\ : \mathsf{Circ}(\mathtt{qubit} \otimes \mathtt{qubit}, \mathtt{bit} \otimes \mathtt{qubit})}$$

Boxes



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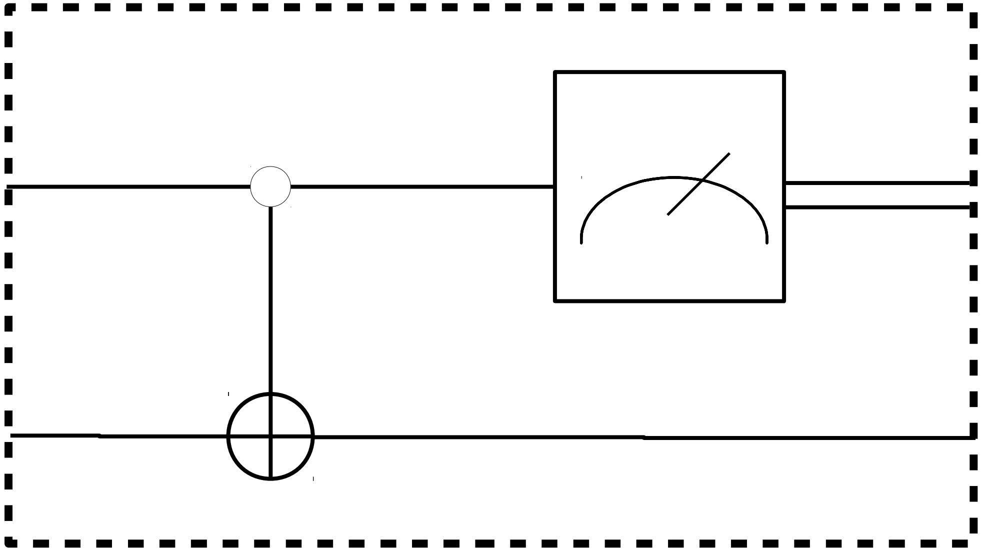
Boxes



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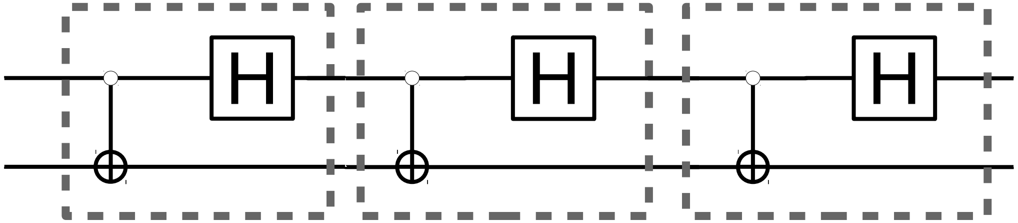
n-ary Composition



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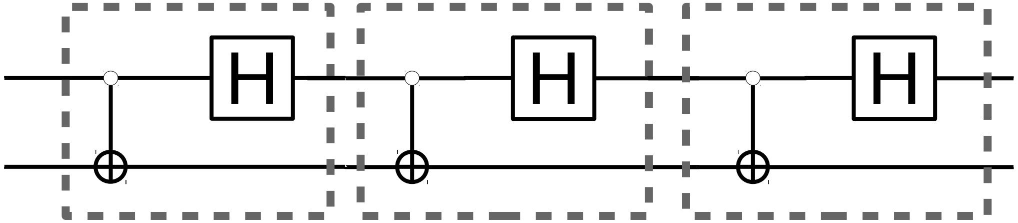
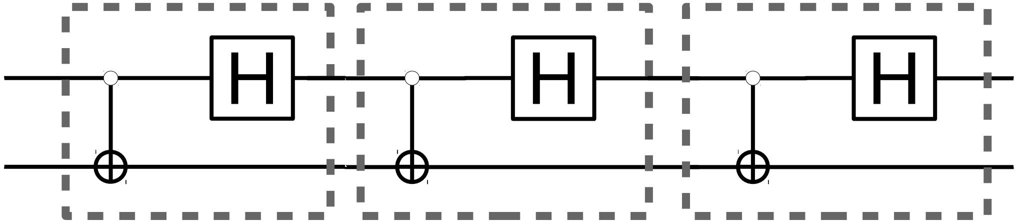


```
inSeqN (n : Nat) (c : Circ(W,W))
                     : Circ(W,W) =
  case n of
```

n-ary Composition



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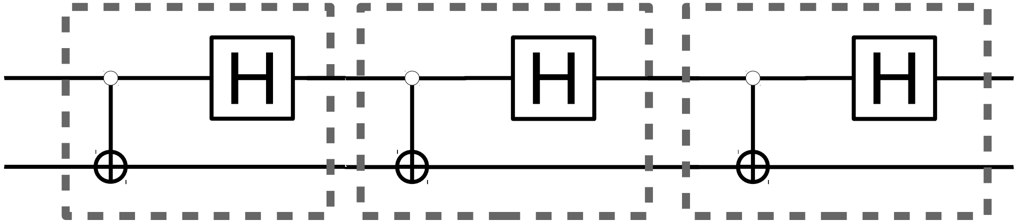

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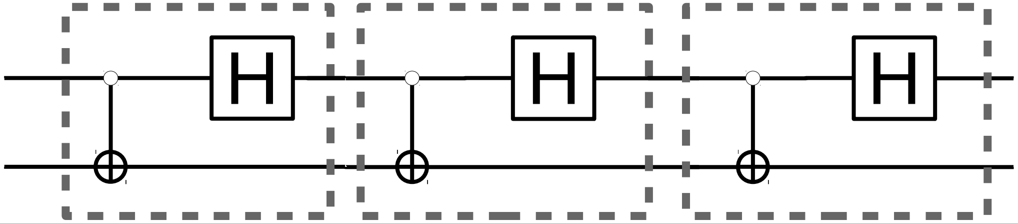

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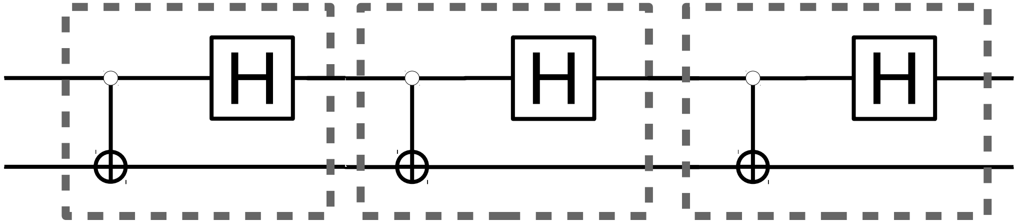

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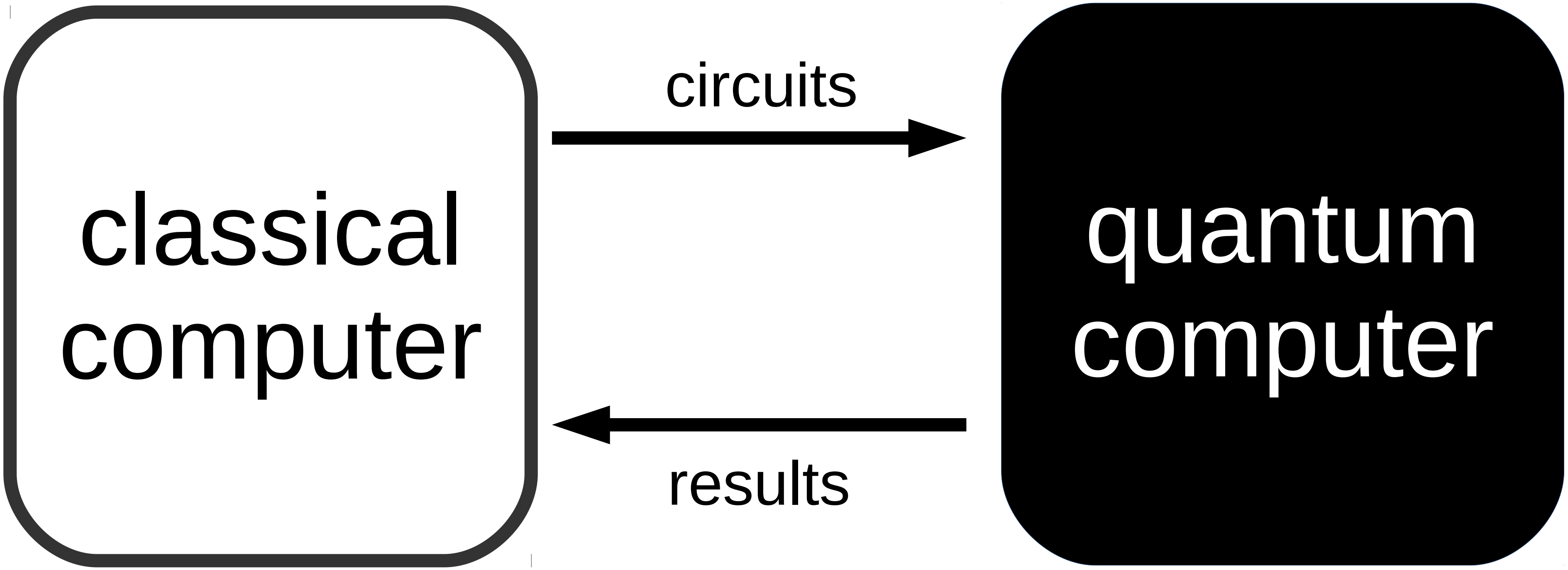
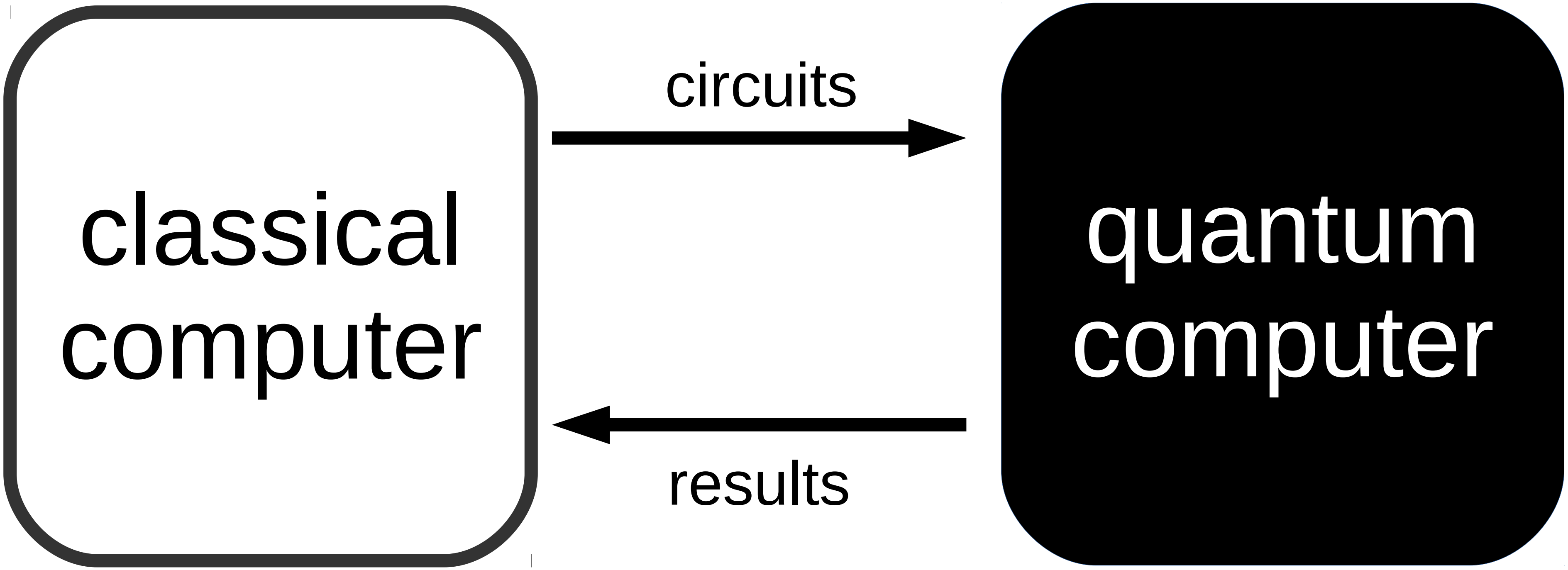
                w' ← unbox c w;
                unbox (inSeqN m) w'
```

Communication



Running Circuits



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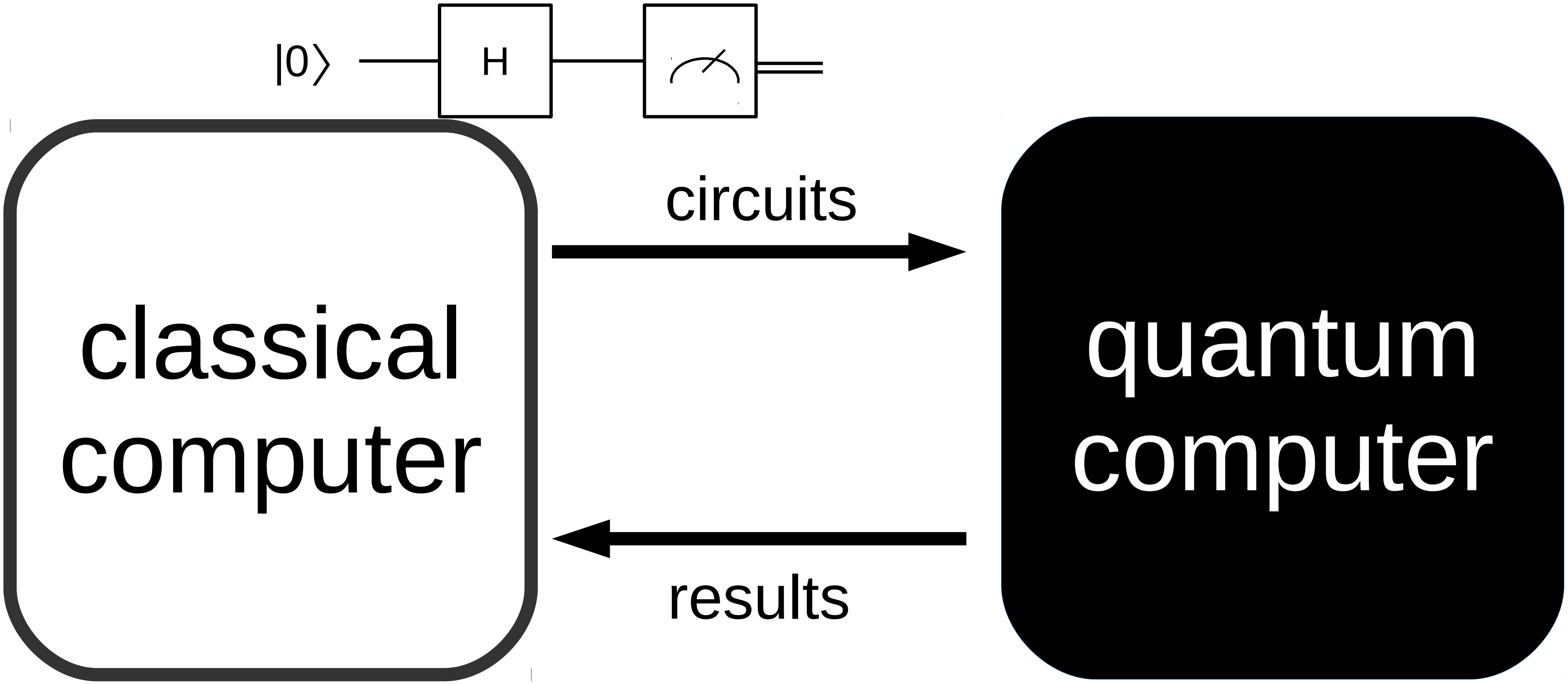
quantum computer

Running Circuits



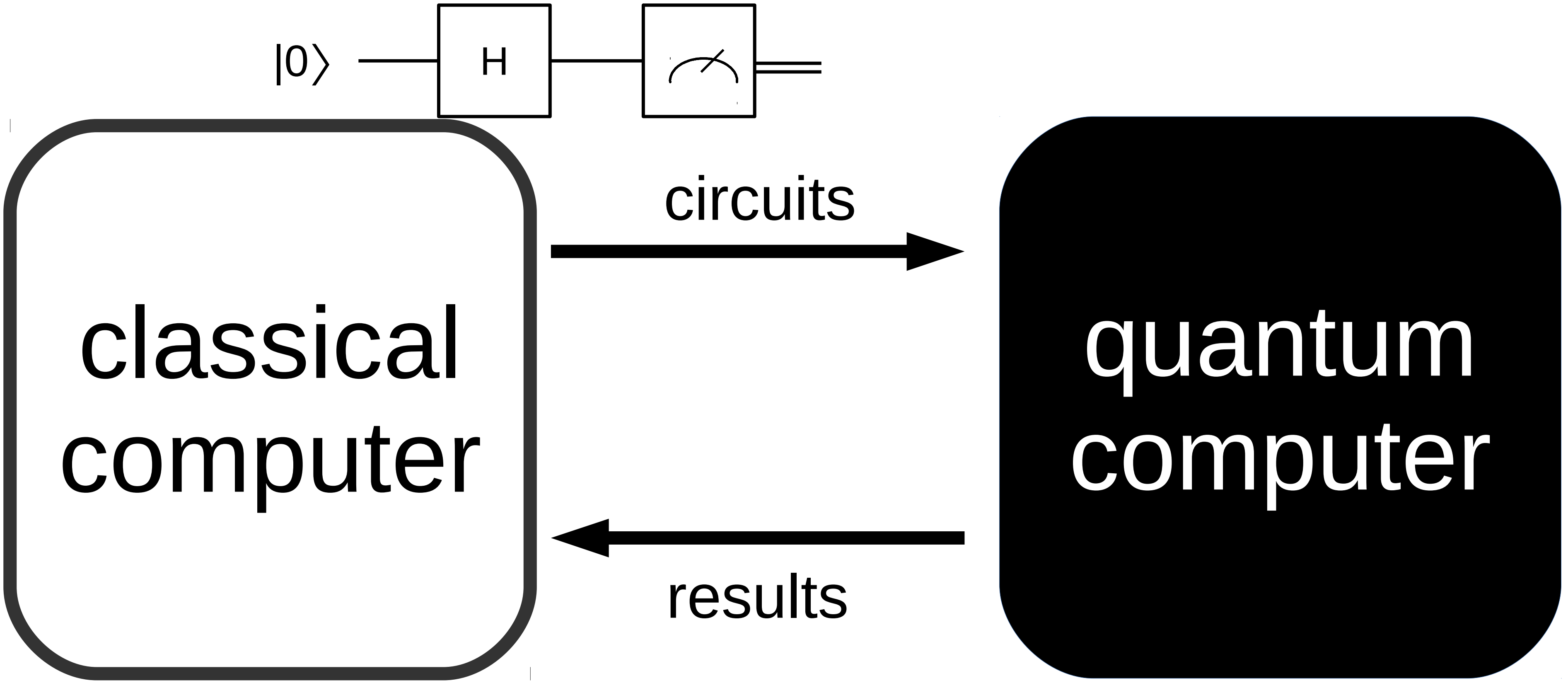
circuits

classical computer

quantum computer

results

Running Circuits



classical computer

circuits

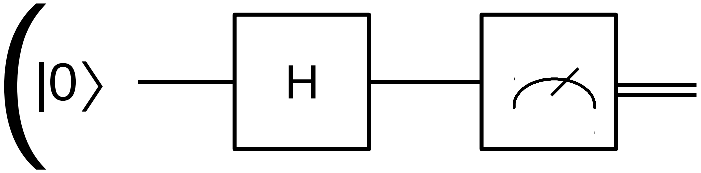
quantum computer

results

$$\left\{ \begin{array}{ll} \text{F} & \text{with probability } 1/2 \\ \text{T} & \text{with probability } 1/2 \end{array} \right\}$$

Running Circuits

$$\frac{\Gamma; \cdot \vdash C : \mathtt{bit}}{\Gamma \vdash \mathsf{run}\ C : \mathsf{Bool}}$$

$\mathsf{run}\Big($ 

$\Big) = \left\{ \begin{array}{ll} \mathrm{F} & \text{with probability } 1/2 \\ \mathrm{T} & \text{with probability } 1/2 \end{array} \right\}$

Dynamic Lifting

Dynamic Lifting

```
b ← gate meas q
```

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if b then ...
     else ...
```

Dynamic Lifting

wire name

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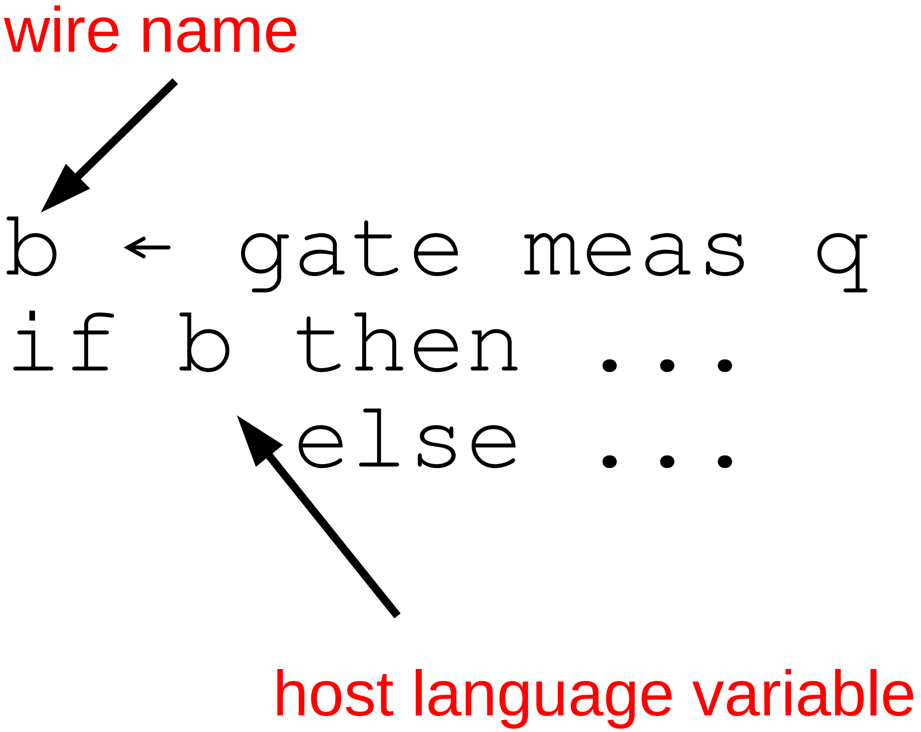
Dynamic Lifting



wire name

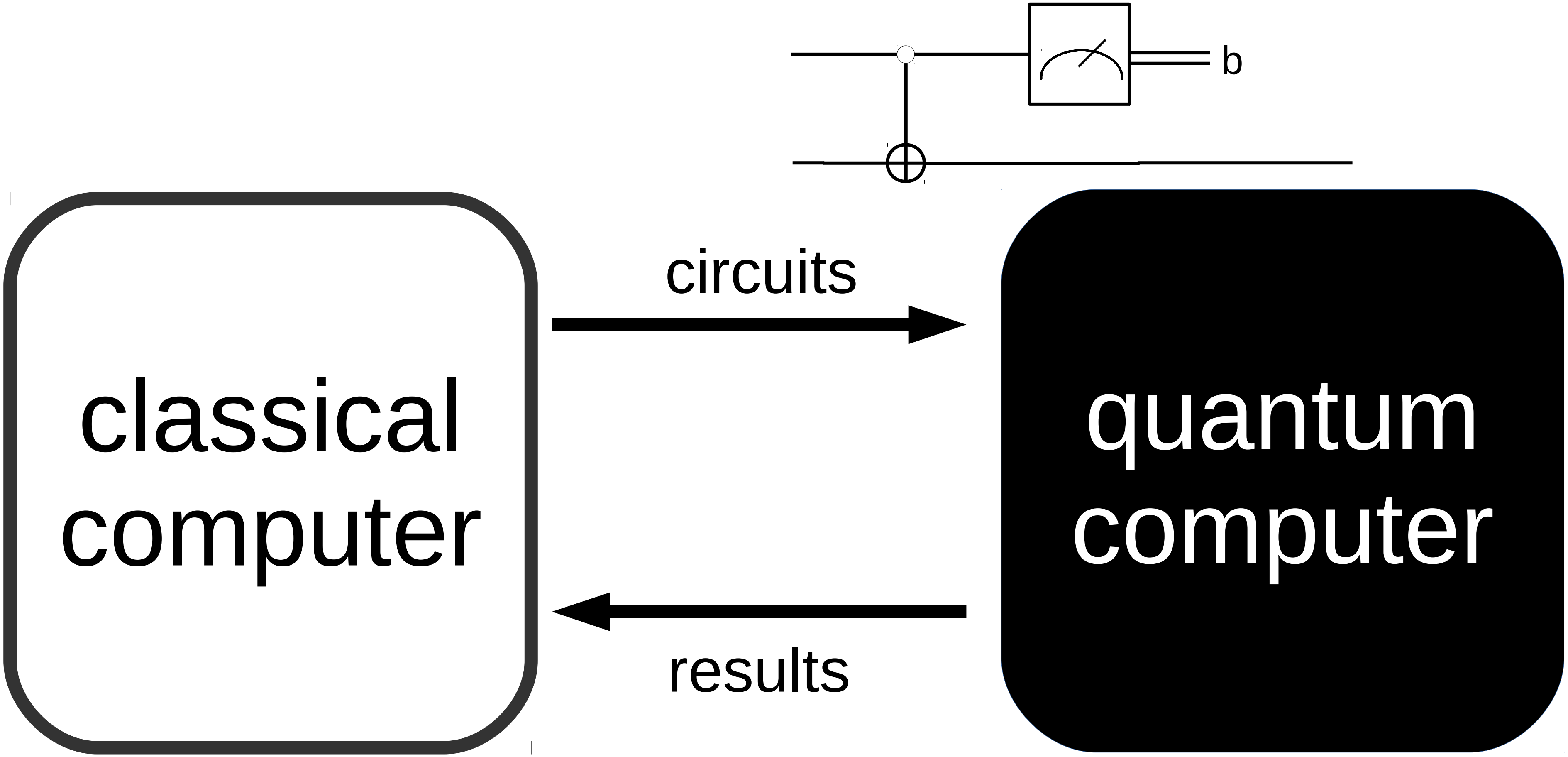
```
b ← gate meas q
if b then ...
     else ...
```

host language variable

Dynamic Lifting

```
b ← gate meas q
x ⇐ lift b;
unbox (if x then ...
            else ...) q'
```

Dynamic Lifting



classical computer

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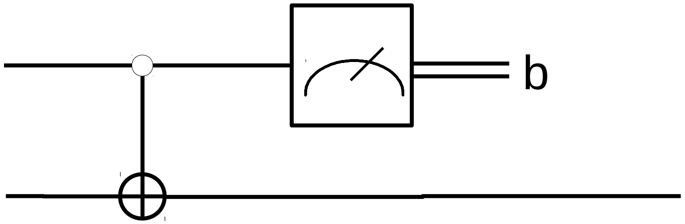
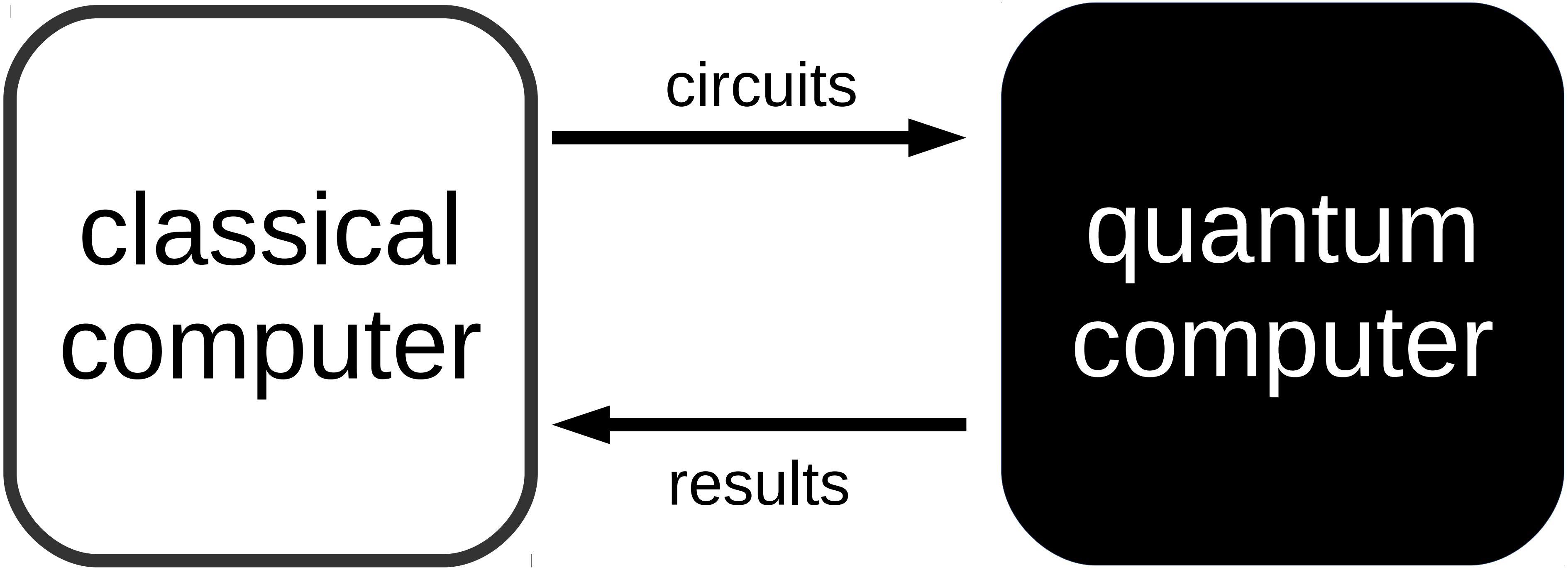
Dynamic Lifting





circuits

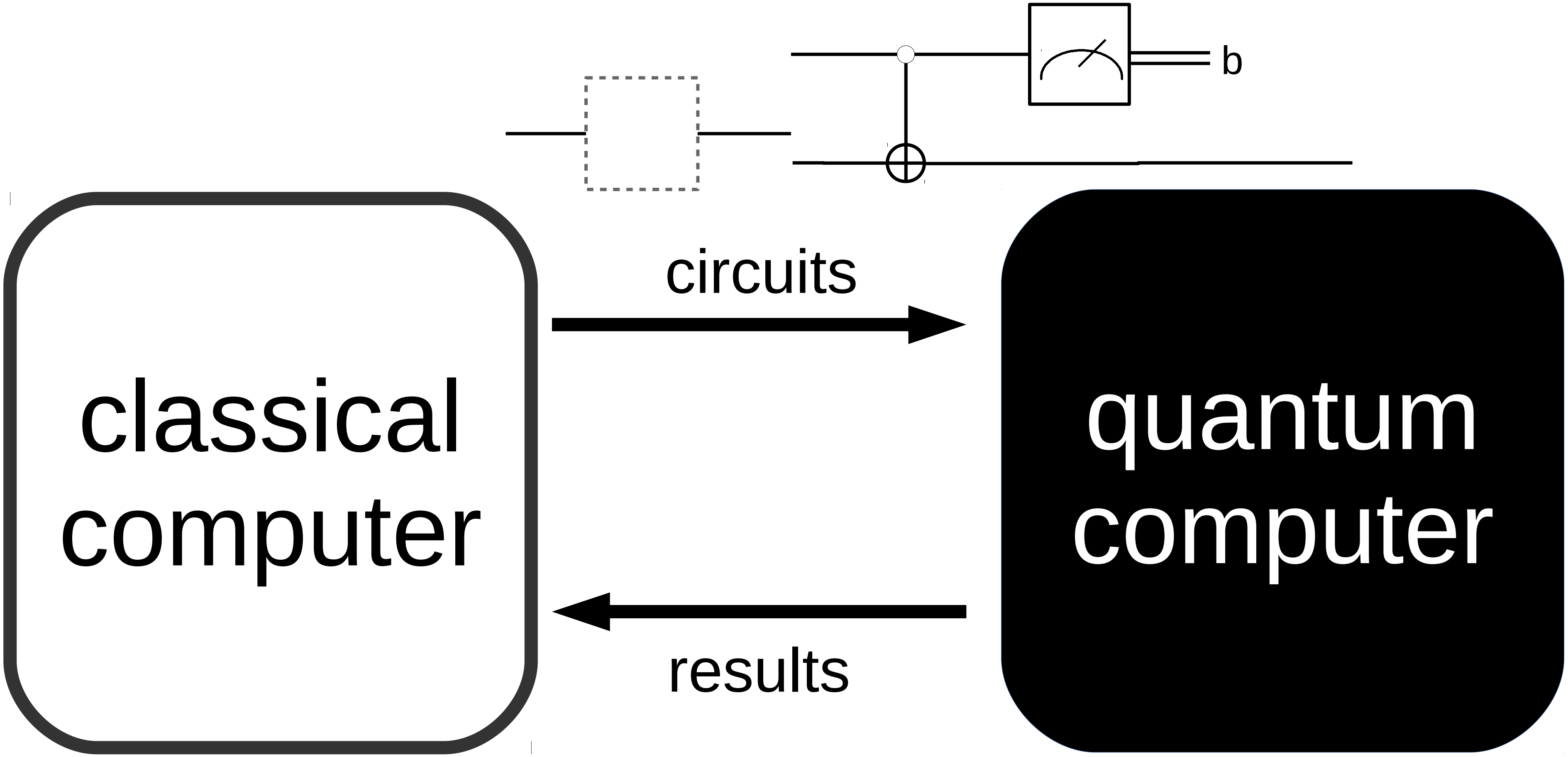
classical computer

quantum computer

results

$$\left\{ \begin{array}{ll} 0 & \text{with probability } 1/2 \\ 1 & \text{with probability } 1/2 \end{array} \right\}$$

Dynamic Lifting



circuits

classical computer

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Summary

```
C ::= output p
    | p' ← gate g p; C
    | p  ← C; C'
    | unbox t p
    | x ⇐ lift p; C

t ::= …
    | box p ⇒ C
    | run C
```

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In the paper

In the paper

- operational semantics of circuits

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- **operational semantics of circuits**
  - proof of type safety
  - proof of strong normalization

In the paper

- **operational semantics of circuits**
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  - proof of strong normalization
- **denotational semantics of circuits**

Denotational Semantics

Selinger 2004

quantum computations

=

superoperators over density matrices

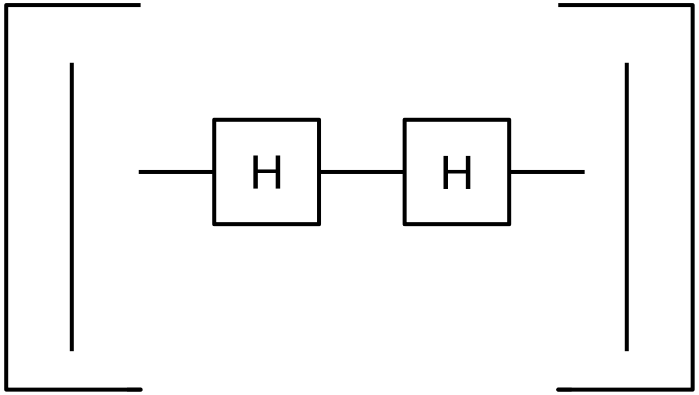
Denotational Semantics

Selinger 2004

quantum computations

=

superoperators over density matrices



$$\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$$

In the paper

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- **operational semantics of circuits**
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In the paper

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  - proof of soundness

In the paper

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- **dependently-typed circuits**

Dependent types

```
qubits (n : Nat) =
  case n of
  | 0   => 1
  | m+1 => qubit⊗(qubits m)
```

Dependent types

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qubits (n : Nat) =
  case n of
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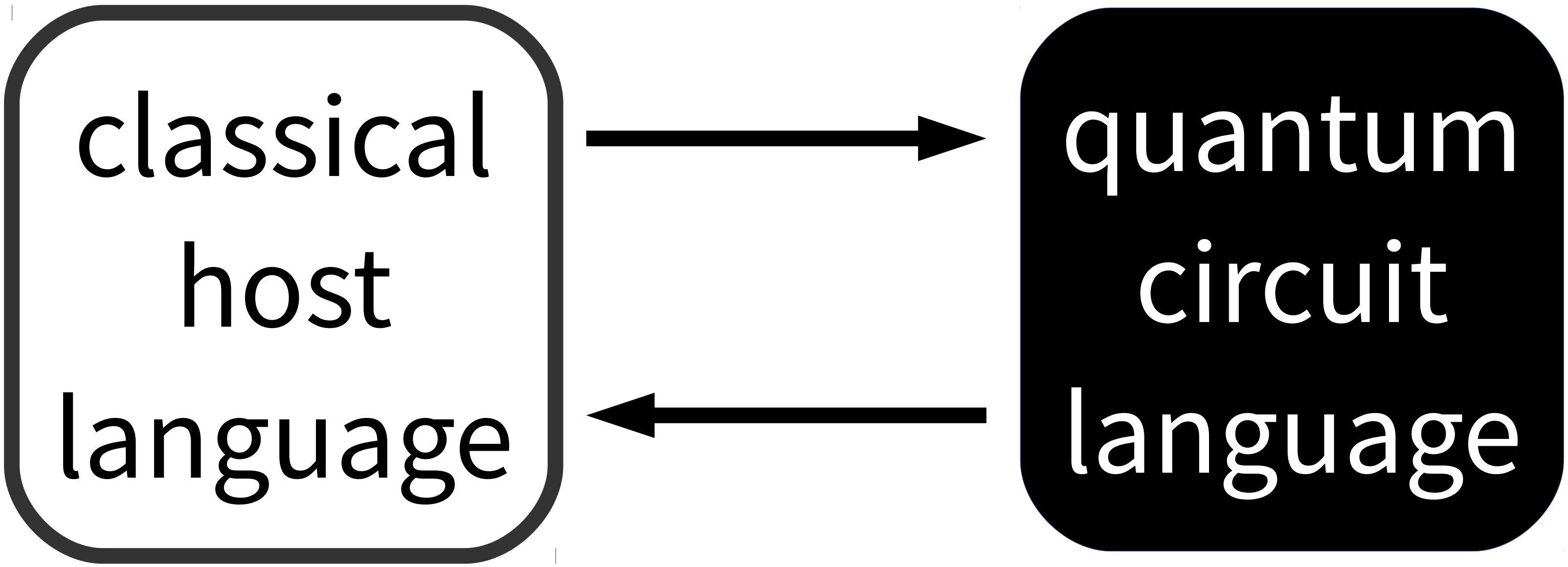
fourier : ∀(n:nat).
     Circ(qubits n, qubits n)
```

In the paper

- **operational semantics of circuits**
  - proof of type safety
  - proof of strong normalization
- **denotational semantics of circuits**
  - proof of soundness
- **dependently-typed circuits**
- **...and more!**

Thank you! Questions?



classical host language

quantum circuit language

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Normal Forms

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```

```
N ::= output p
    | p' ← gate g p; N
    | x ⇐ lift p; C
```

Types

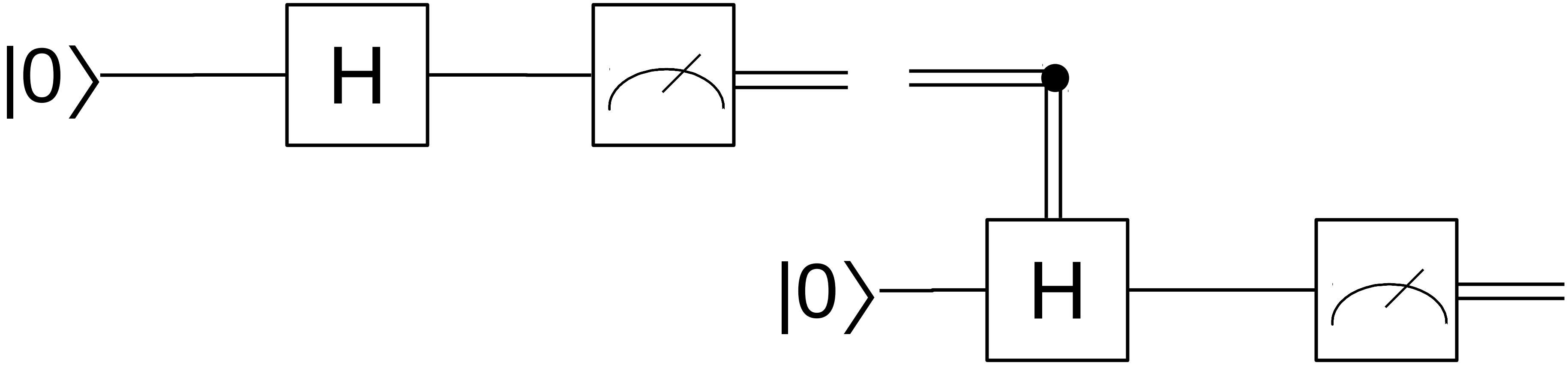
$$t ::= \cdots \mid \mathsf{Circ}(W_1, W_2)$$

Types

$$W ::= \texttt{qubit} \mid \texttt{bit} \mid 1 \mid W_1 \otimes W_2$$

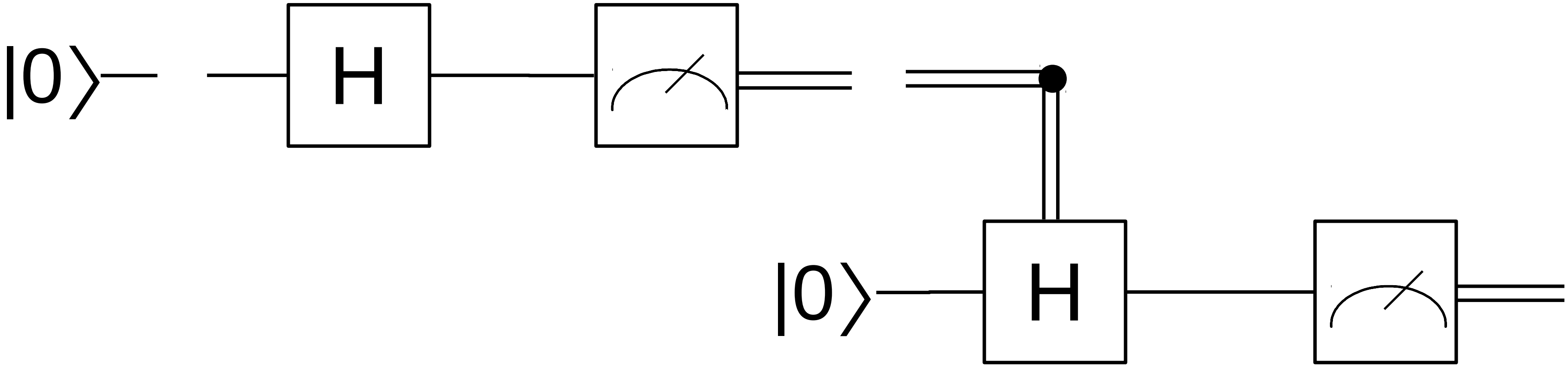
$$t ::= \cdots \mid \mathsf{Circ}(W_1, W_2)$$

Composition is associative



```
b ← (q  ← gate new0 ();
     q' ← gate H    q ;
     b' ← gate meas q';
     output b');
r ← new0 ();
...
```

Composition is associative



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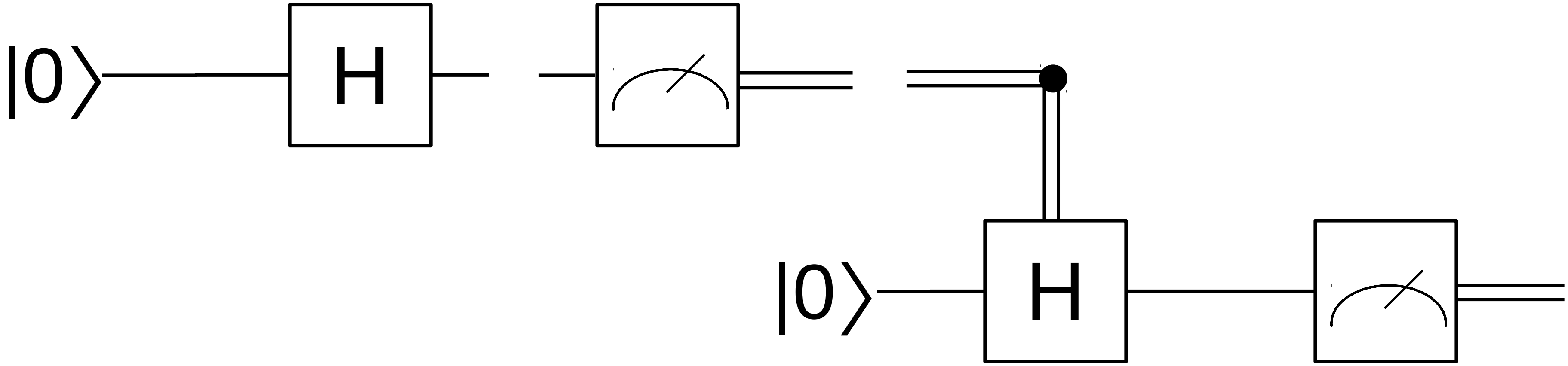

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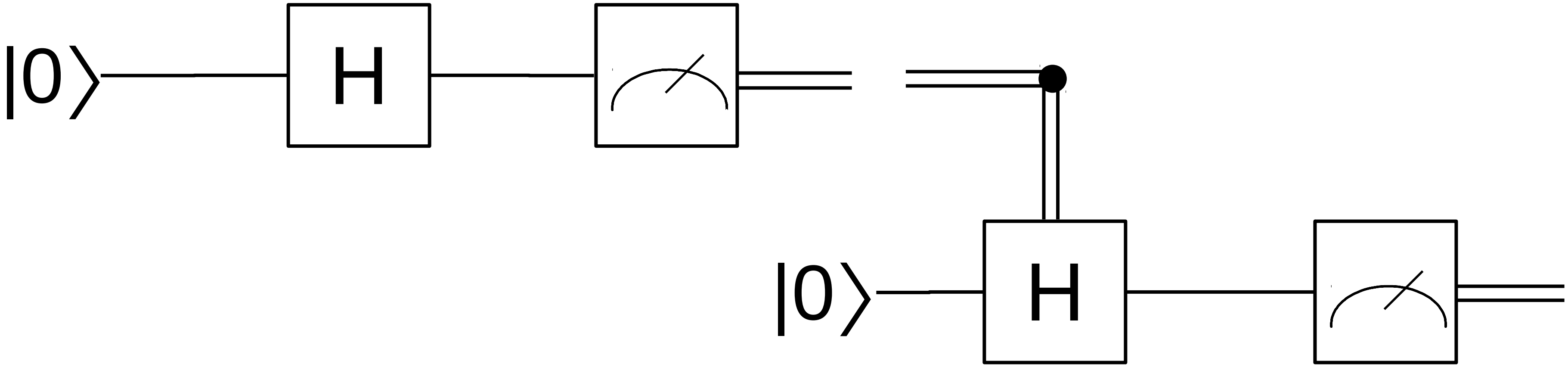

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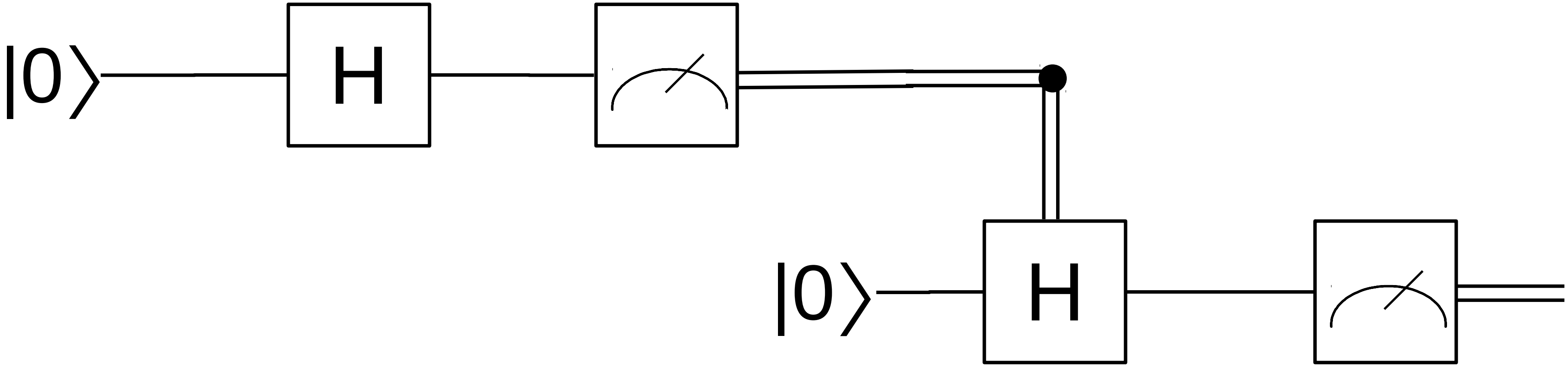

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