Tracking Errors through Types in Quantum Programs

Kesha Hietala, *Robert Rand*, Michael Hicks

Programming Languages for Quantum Computing

PLanQC 2020

Overview

- Errors, particularly those introduced by gate application, will be prevalent in near-term quantum machines, so need to be taken into account when writing programs.
- It might be useful to have a notion of errors at the *programming language level*
- We have designed a simple type system to track errors in quantum programs, implemented in **QWIRE**

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- A linear type system for enforcing no-cloning
- A denotational semantics in terms of density matrices

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Inductive Circuit (W : WType) : Type :=
| output : Pat W -> Circuit W
| gate   : Gate W1 W2 -> Pat W1 ->
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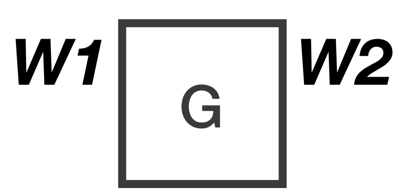

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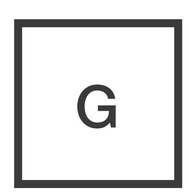
W1  *W2*

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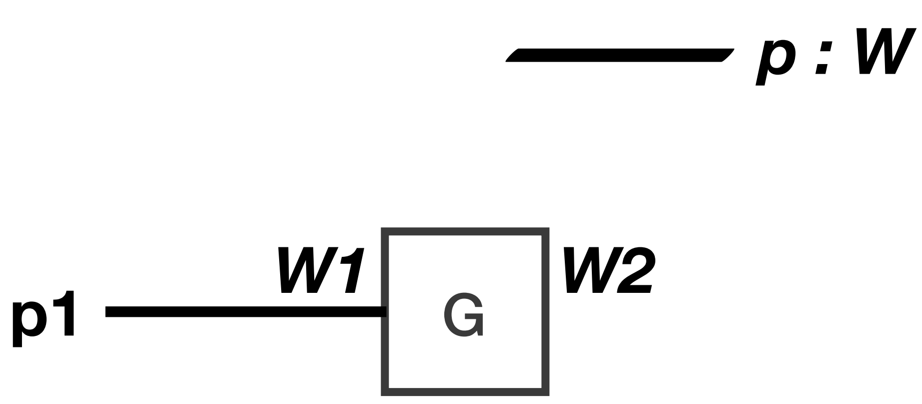

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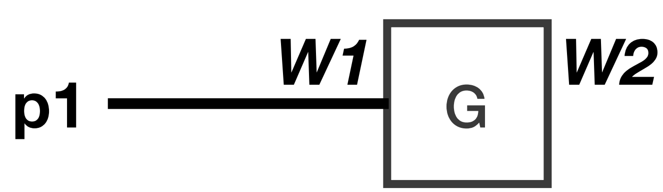

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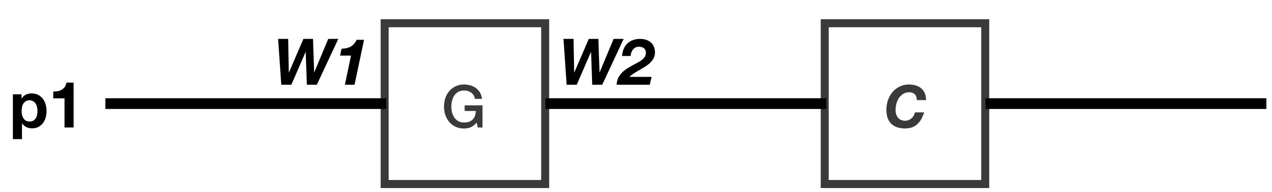

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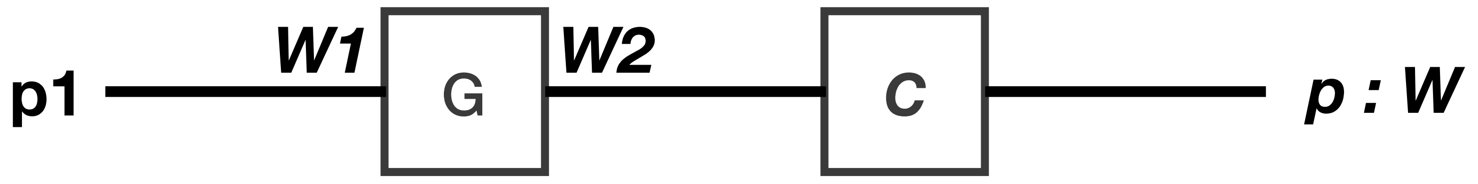

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 | meas    : Gate Qubit Bit
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$$|0\rangle_E = (\,|000\rangle + |111\rangle) \otimes (\,|000\rangle + |111\rangle) \otimes (\,|000\rangle + |111\rangle)$$

$$|1\rangle_E = (\,|000\rangle - |111\rangle) \otimes (\,|000\rangle - |111\rangle) \otimes (\,|000\rangle - |111\rangle)$$

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Error Prone Gates

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Inductive Gate : nat -> WType -> WType -> Set :=
 | U       : (U : Unitary k W) ->
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 | init0   : Gate ϵ_i One (Qubit ϵ_i)
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 | meas    : Gate ϵ_m (Qubit n) Bit
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Fault Tolerance

Gottesman (2009): An Introduction to Quantum Error Correction and Fault-Tolerant Quantum Computation

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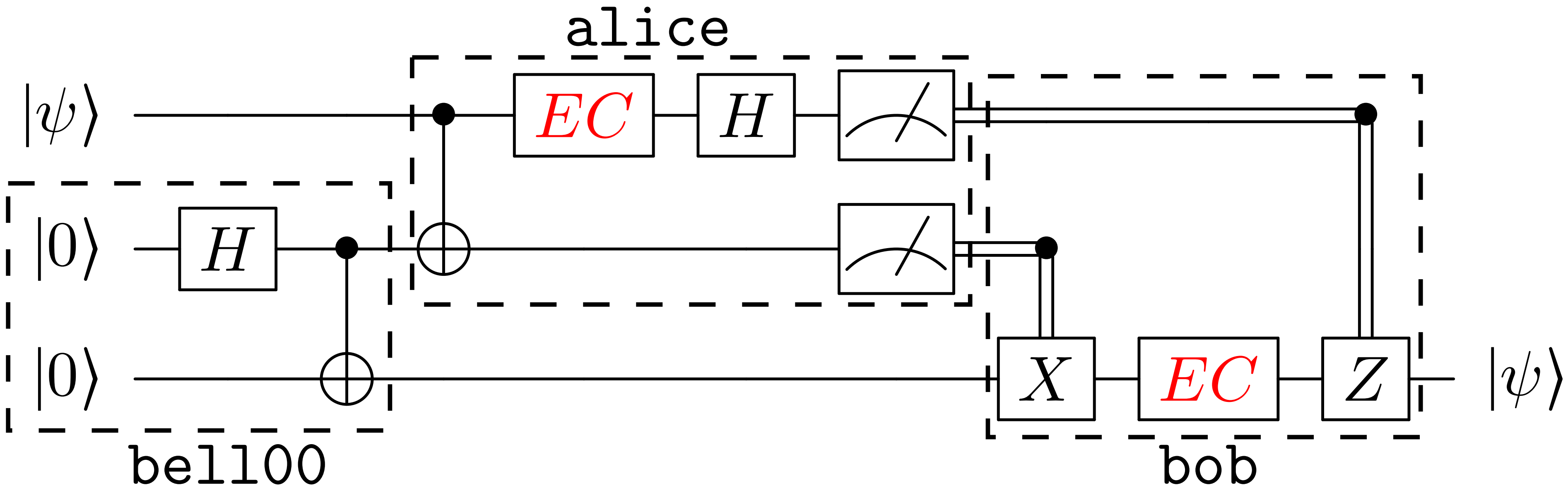
- We benefit from *linearity* — we cannot introduce an error *onto* an existing qubit, but only produce a new qubit with some number of errors
- However, *checking* fault tolerance is orthogonal to checking linearity.

Typechecking

```
Inductive Types_Circuit :  OCtx -> nat -> Circuit W -> Set :=
 | types_gate : forall {Γ Γ1 Γ1' w1 w2 w}
                (f : Pat w2 -> Circuit w)
                (k t : nat) {p1 : Pat w1} {g : Gate k w1 w2},
                Γ1 ⊢ p1 :Pat ->
                k + sum_err W1 <= t ->
               (forall Γ2 Γ2' (p2 : Pat w2)
                  {pf2 : Γ2' == Γ2 · Γ},
                  Γ2 ⊢ p2 :Pat -> Types_Circuit Γ2' t (f p2)) ->
               forall {pf1 : Γ1' == Γ1 · Γ},
               Types_Circuit Γ1' t (gate g p1 f).
```

Typechecking



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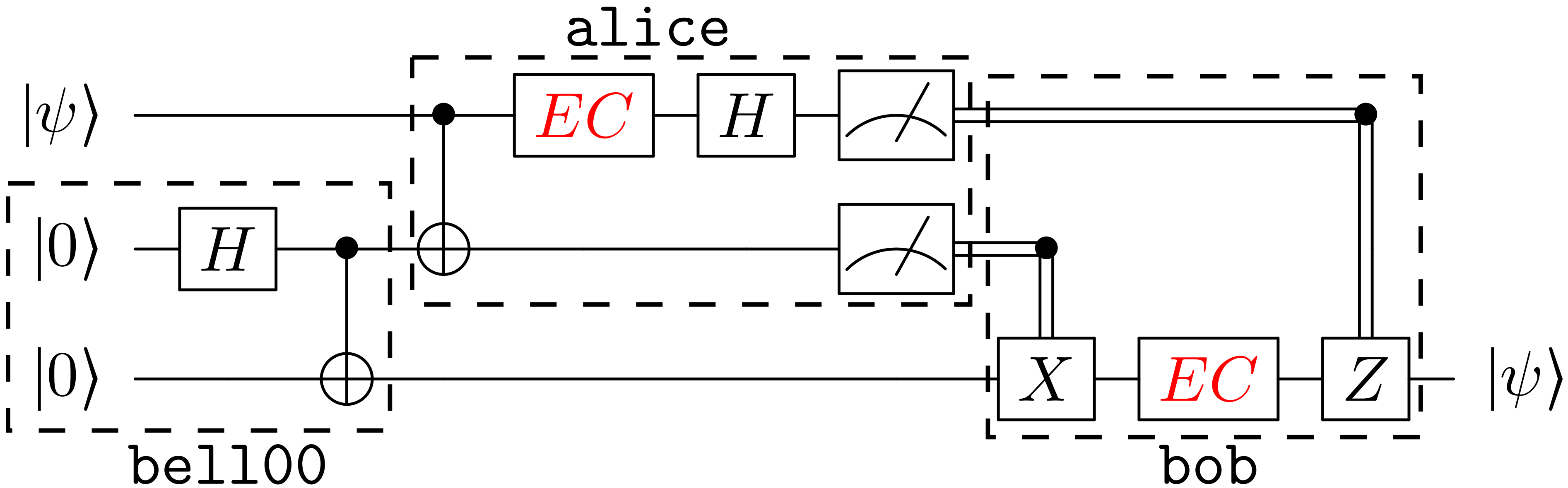


```
Definition bell : Box One (Qubit 2 ⊗ Qubit 2) := …
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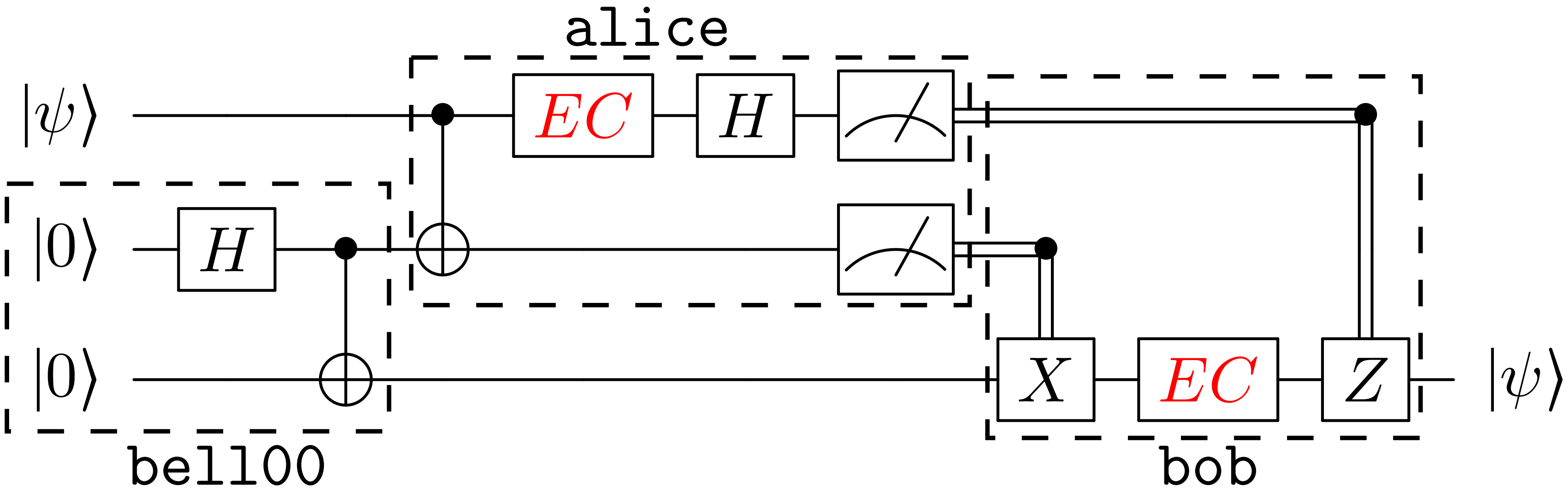


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Definition bell : Box One (Qubit 2 ⊗ Qubit 2) := …

Definition alice : Box (Qubit 0 ⊗ Qubit 2) (Bit ⊗ Bit) := …
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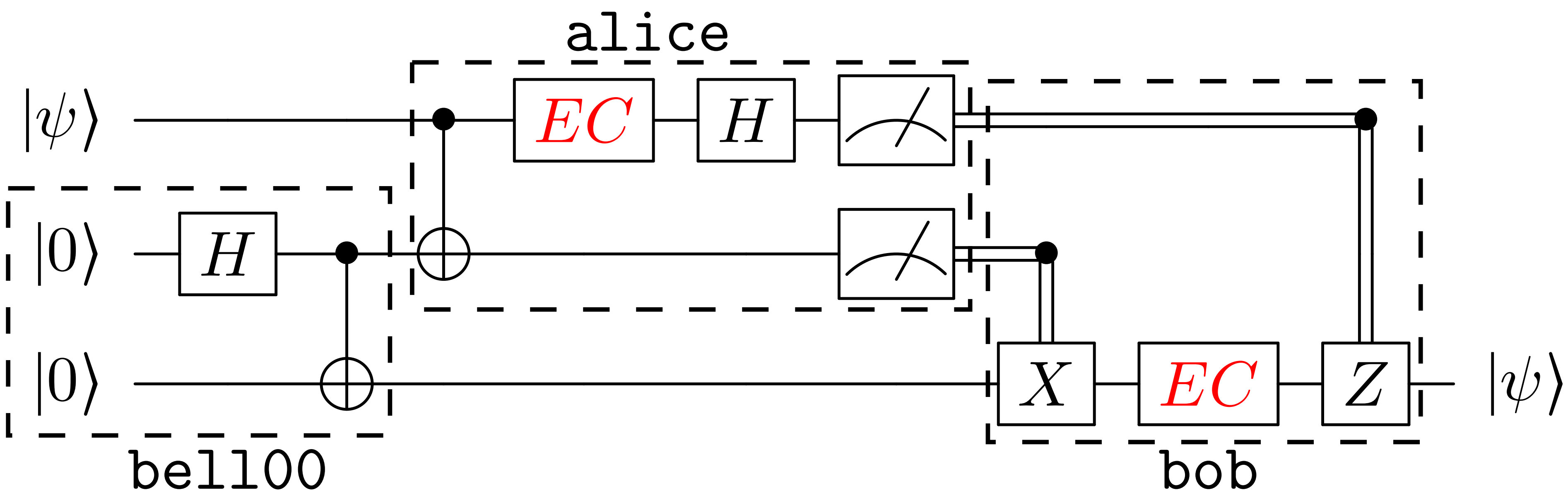


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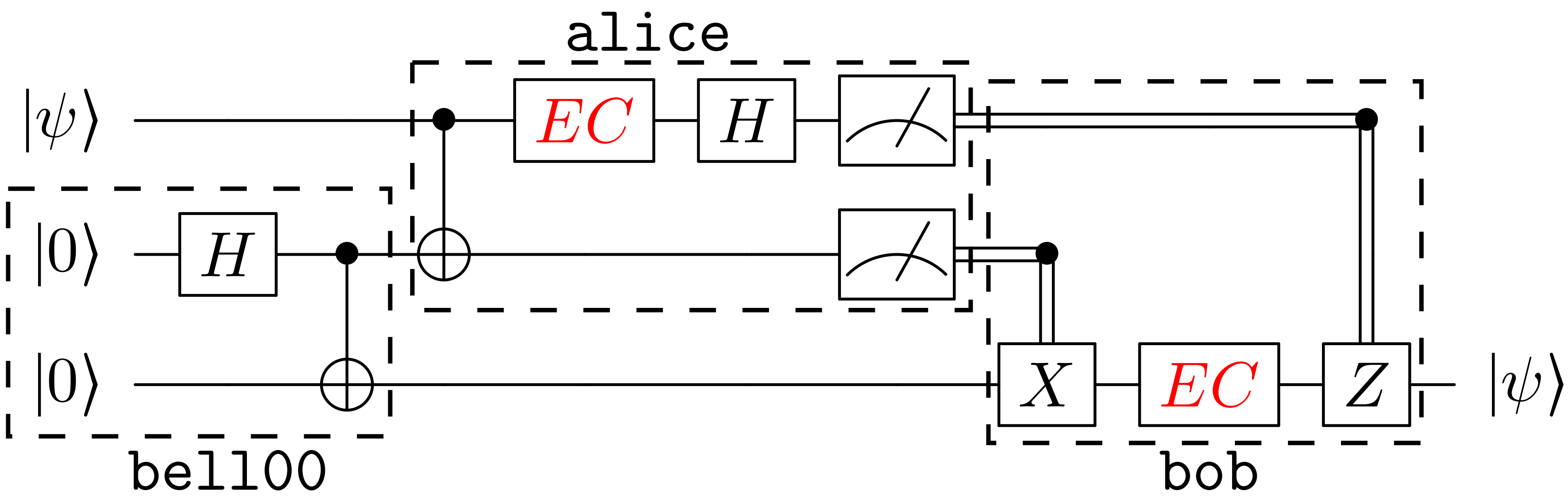


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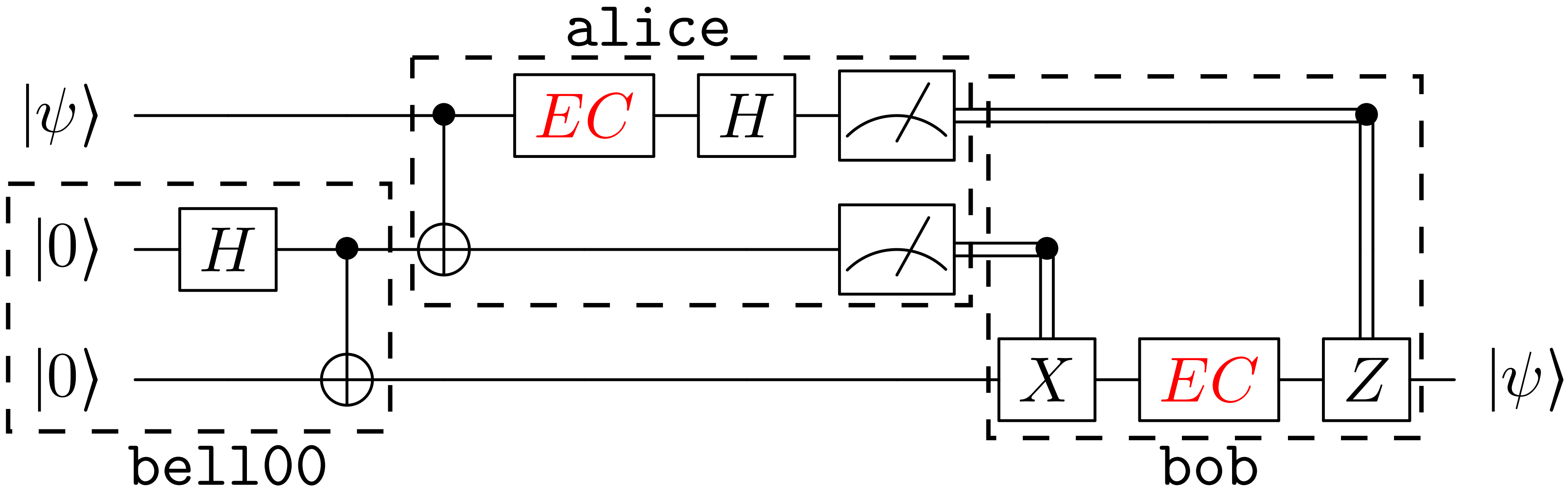


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Lemma teleport_EC_WT : Typed_Box teleport 3.
Proof. type_check. Qed.
```
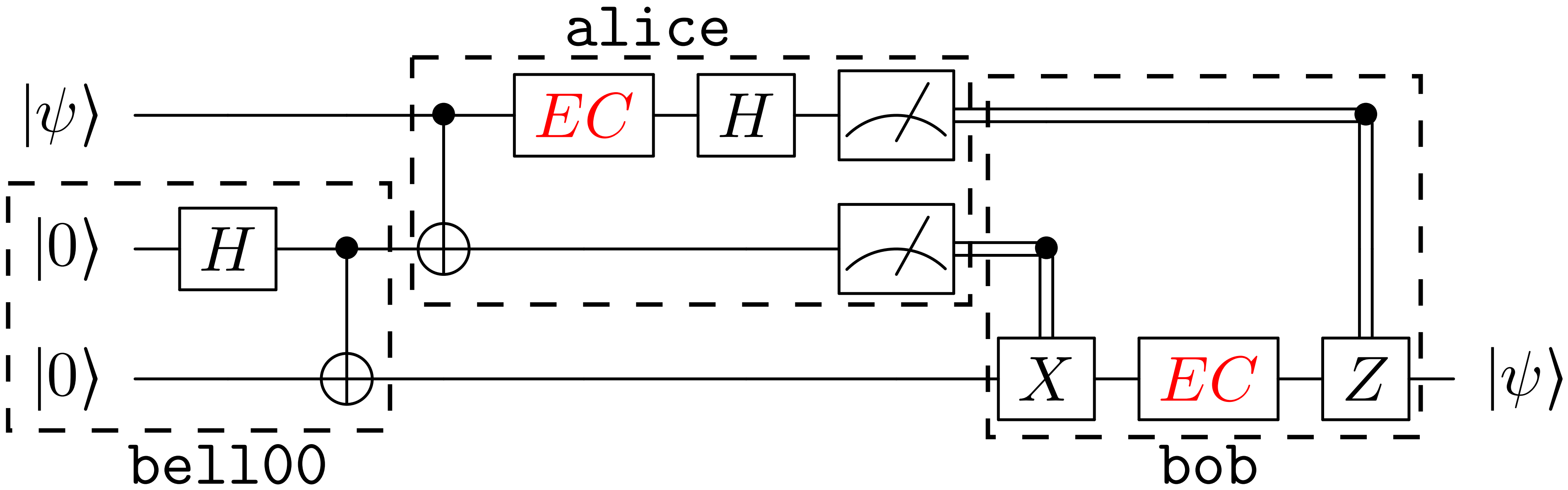

Type Inference



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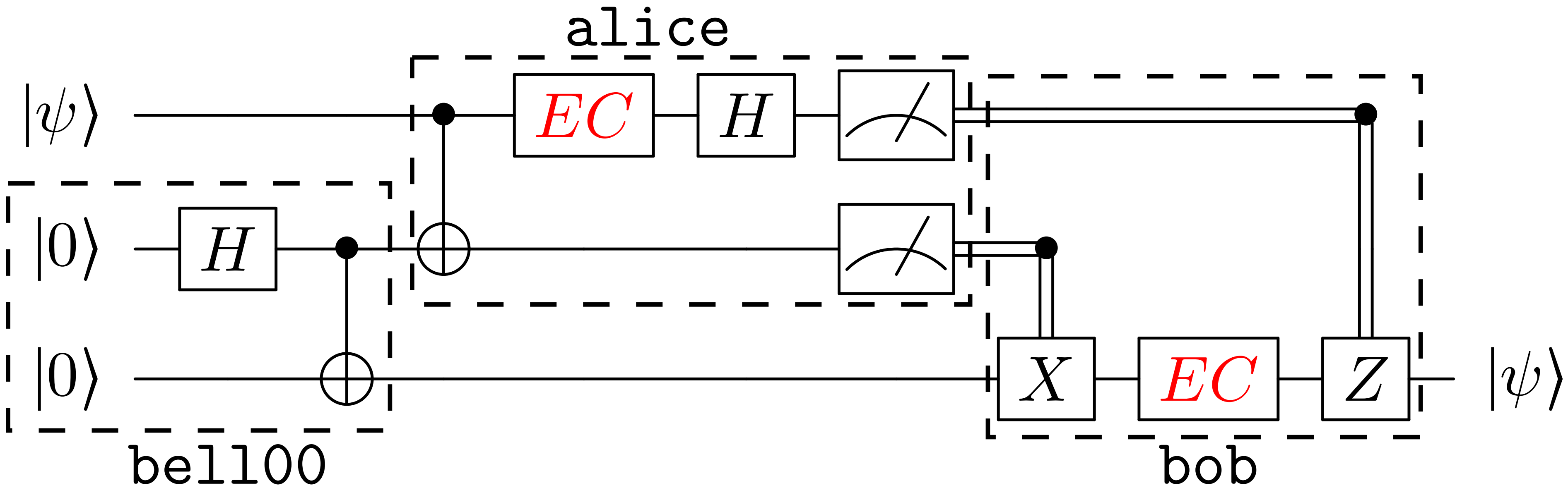


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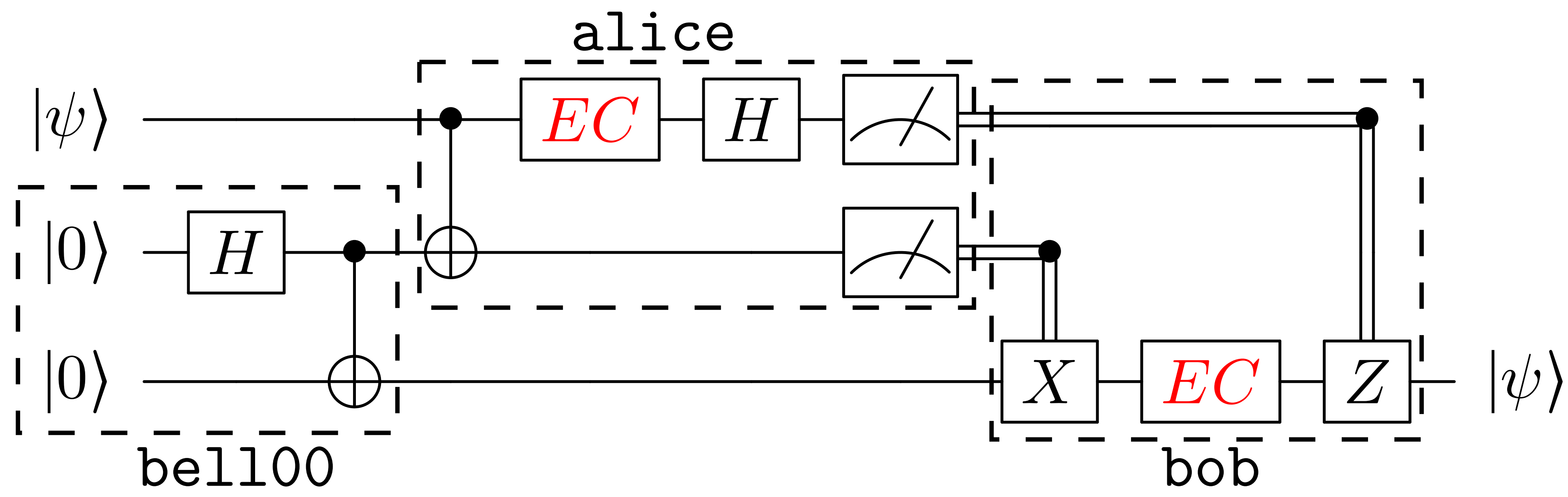

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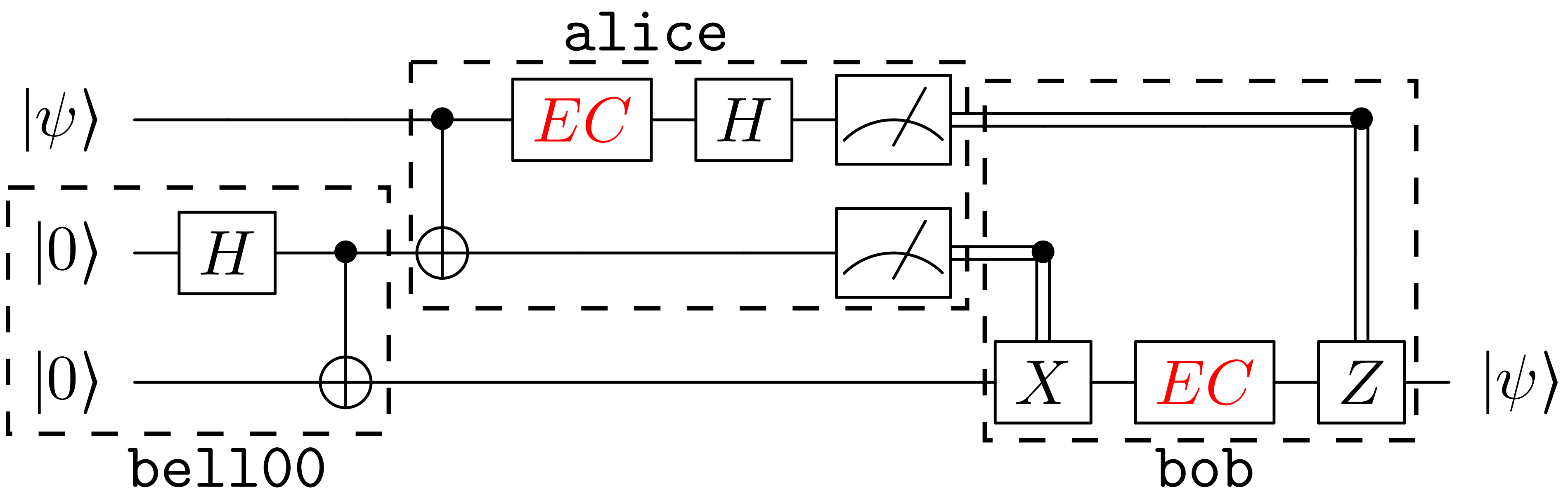


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Definition bell := …                          : Box One (Qubit 2 ⊗ Qubit 2)
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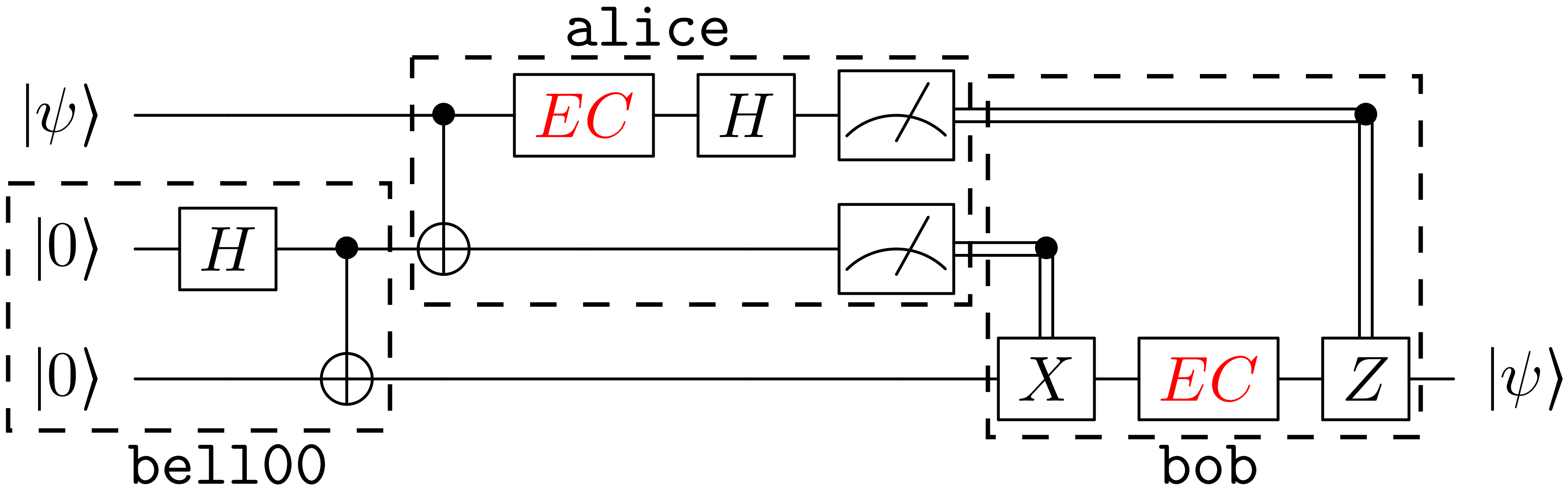

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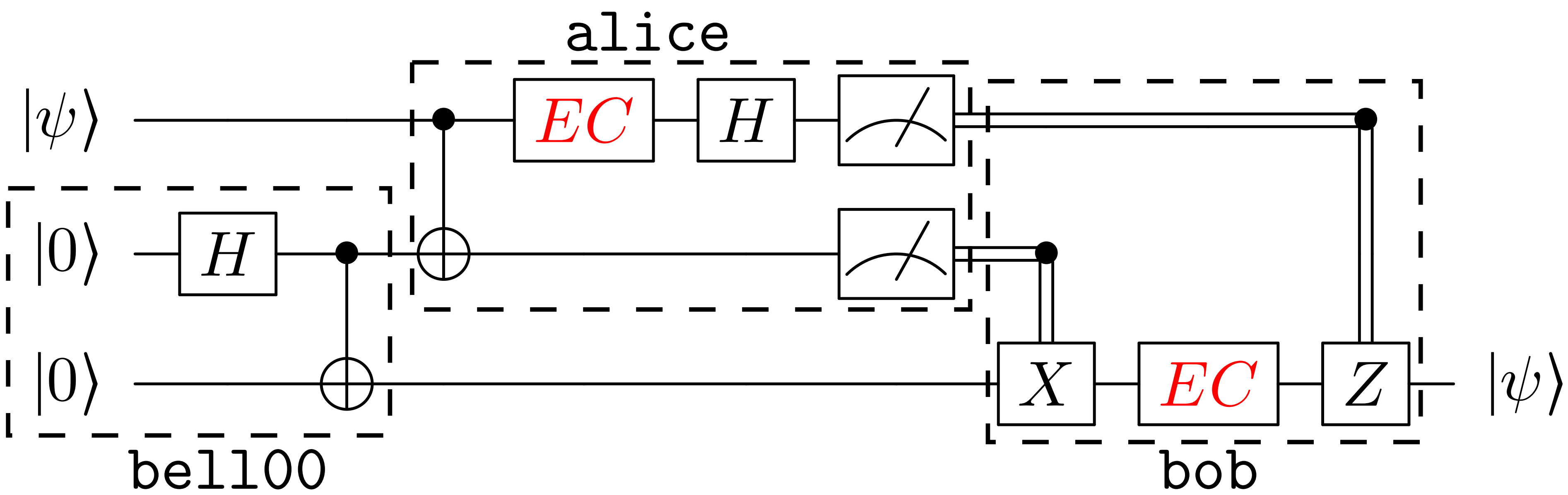

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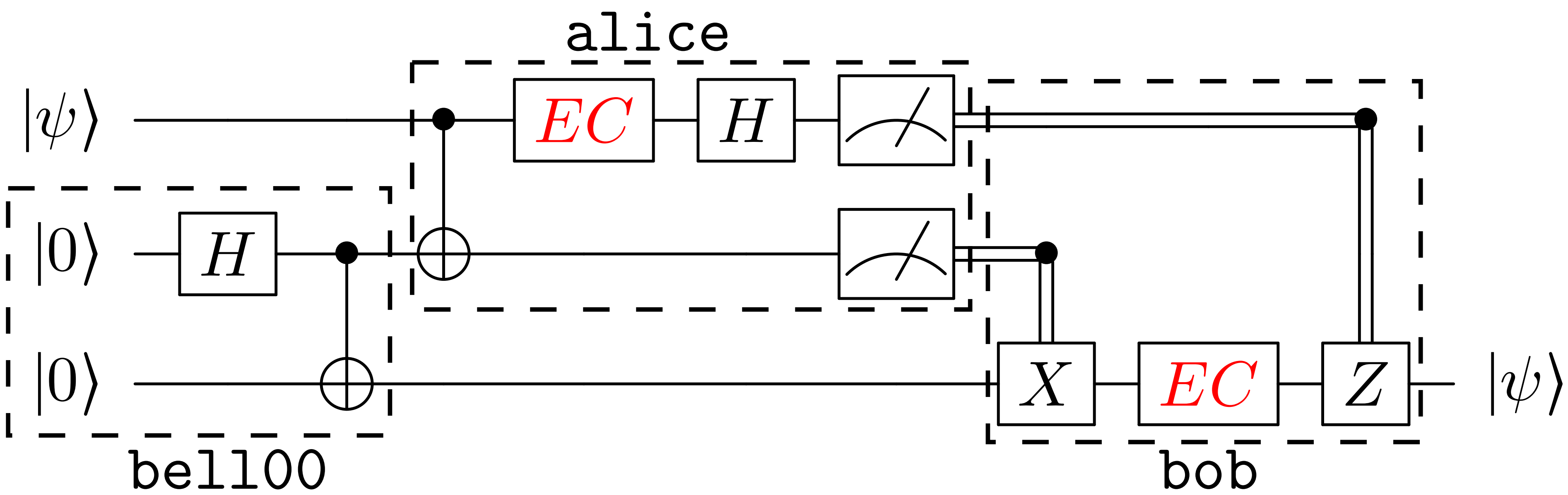

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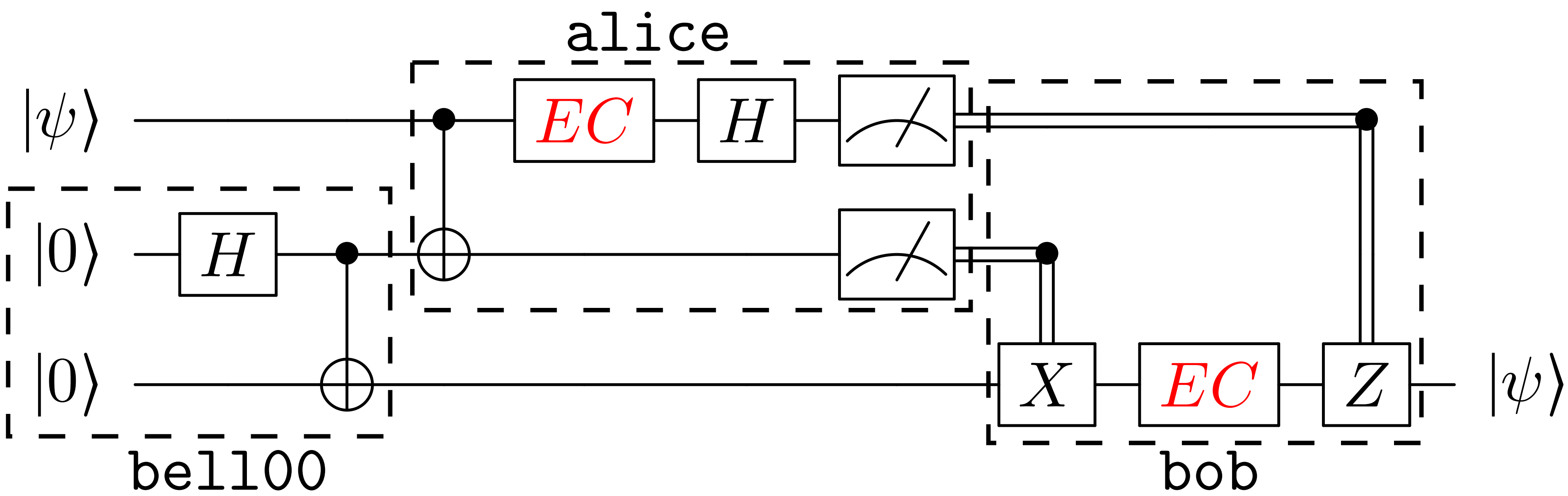

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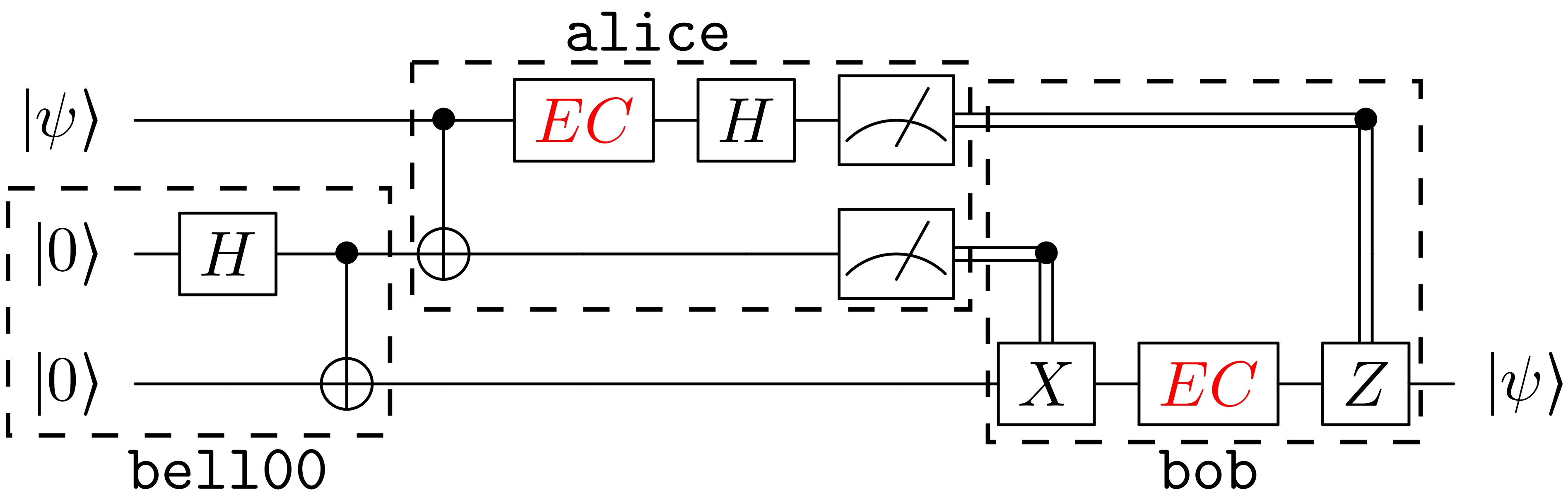


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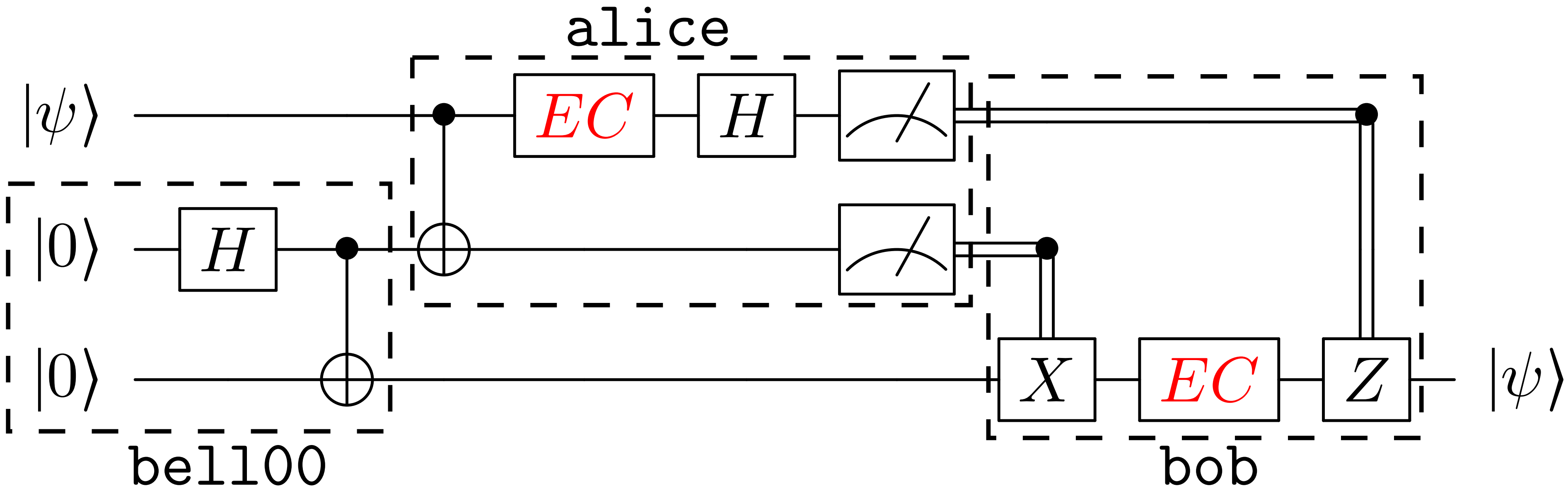

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Definition teleport := …                               : Box (Qubit 0) (Qubit 1)
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Questions

- How can we reflect our error handling back into QWIRE's denotational semantics?
- What about errors due to decoherence, that aren't captured by gate application?
- What are better ways of handling errors in a type system or broader reasoning system?