1 Number Theory

Problem 1: Suppose that $\text{GCD}(a, b, c) = 1$ where $\text{GCD}$ is extended to three integers in the natural way. Are there necessarily two numbers from $\{a, b, c\}$ that are relatively prime? Give a proof.

Problem 2: Define $-[a]_m = [-a]_m$ where $m \neq 0$. Is this negation a valid function? Give a proof.

Problem 3: Suppose $a \equiv_m b$ where $m$ is a composite number greater than 2. Demonstrate that there are 3 distinct values of $m$ such that $a \mod m = b \mod m$.

2 Induction

Problem 4: Prove the commutativity of addition using only the axioms of Peano arithmetic. You may want to state and prove an intermediate lemma.