CMSC 27100-3	Due: Friday, October 15, 2021

Problem Set 2

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1 Predicate Calculus II

Problem 1: Assume $\forall x, Px \wedge Qx$. Prove that $\forall x, Qx \vee Px$.

Problem 2: Assume $\forall x, \forall y, Px \rightarrow Qy$ and $\exists z, \neg Qz$. Prove that $\exists x, \neg Px$.

2 Functions and relations

Problem 3: Write a bijection f between the sets

$$\{(x, y) \in \mathbb{N} \times \mathbb{N} \mid x = y + 1 \vee y = x + 1)\}$$

and $\mathbb{Z}$. Also write the inverse function.

Problem 4: Let R be a relation over sets where $(A, B) \in R$ means "there is a bijection from A to B". For each of the questions below include a proof (a proof sketch suffices for 3).

1. Is R reflexive, irreflexive or neither?
2. Is R symmetric, asymmetric, antisymmetric or none?
3. Is R transitive?

Problem 5: Let R be the *divides* relation restricted to pair of naturals ($R \subseteq \mathbb{N} \times \mathbb{N}$). For each of the questions below include a proof.

1. Is R reflexive, irreflexive or neither?
2. Is R symmetric, asymmetric, antisymmetric or none?
3. Is R transitive?