1 Predicate Calculus II

Problem 1: Assume $\forall x, Px \land Qx$. Prove that $\forall x, Qx \lor Px$.

Problem 2: Assume $\forall x, \forall y, Px \rightarrow Qy$ and $\exists z, \neg Qz$. Prove that $\exists x, \neg Px$.

2 Functions and relations

Problem 3: Write a bijection $f$ between the sets 
\[ \{(x, y) \in \mathbb{N} \times \mathbb{N} \mid x = y + 1 \lor y = x + 1\} \]
and $\mathbb{Z}$. Also write the inverse function.

Problem 4: Let $R$ be a relation over sets where $(A, B) \in R$ means “there is a bijection from $A$ to $B$”. For each of the questions below include a proof (a proof sketch suffices for 3).

1. Is $R$ reflexive, irreflexive or neither?
2. Is $R$ symmetric, asymmetric, antisymmetric or none?
3. Is $R$ transitive?

Problem 5: Let $R$ be the divides relation restricted to pair of naturals ($R \subseteq \mathbb{N} \times \mathbb{N}$). For each of the questions below include a proof.

1. Is $R$ reflexive, irreflexive or neither?
2. Is $R$ symmetric, asymmetric, antisymmetric or none?
3. Is $R$ transitive?