1 Propositional logic

Problem 1: Augustus De Morgan has two laws named after him. The first says that the following two propositions are equivalent:

1. It is not the case that both $P$ and $Q$ are true
2. $P$ is not true or $Q$ is not true

Use a truth table to prove this.

Problem 2: De Morgan’s second law says that if it’s not the case that $P$ or $Q$ is true, then $P$ is not true and $Q$ is not true. Prove this using natural deduction.

Problem 3: De Morgan’s second law also goes in the reverse direction. Prove this via natural deduction or the Incredible Proof Machine (your choice). (You can screenshot or draw the output of the proof machine.)

2 Predicate Calculus I

Problem 4: State the following claims as predicate calculus formulas. Define your predicates and named variables (e.g. “B x y = x is bigger than y”, “e = Ernie”)

1. All dogs go to heaven.
2. Every natural number is either even or not even.
3. Elizabeth is the queen of some country.
4. Nobody is the queen of everything.
5. Ra illuminates everything that does not illuminate itself.